Journal of Positive School Psychology http://journalppw.com
2022, Vol. 6, No. 5, 2346 – 2351

A new mathematics model to find optimal control of Optimization problem

[1]Mazin Kareem Marjan, [2]Dr. Ahmed Sabah Aljilawi

[1]*Ministry of Education, General Directorate of Education in Babylon, mazen.marjan22@gmail.com*
[2]*Department of Mathematics -University of Babylon, aljelawy2000@yahoo.com*

Abstract

In order to achieve the shortest possible final time, solve the problem of optimal control. The problem of mathematics model control criterion was prepared and solved by our group. In addition, we developed a program to improve and display the results of our experiments.

Keywords: Dynamic Optimization, Formulation Strategies, Numerical Differential Equation, Optimal Control, GEKKO (Python).

1. INTRODUCTION

It is referred to as dynamic control when the method of using model predictions to plan an optimized future trajectory for time-varying systems is used (DC). [1,3,] Several terms are frequently used to describe this process, including Model Predictive Control (MPC) and Dynamic Optimization. Using numerical integration to solve dynamic control problems at discrete time intervals, which is analogous to measuring a physical system at specific time points in real time, a method for solving dynamic control problems has been developed. Every time step, the numerical solution is compared to a desired trajectory, and the difference between the two is reduced to the smallest possible value by adjusting parameters in the model that are susceptible to variation. First, the first control action must be completed; then the entire process must be completed again, at the next time instance. Due to the fact that objective targets can change over time and that updated measurements can result in revised parameter or state estimates, the process is repeated in this case.

1-1 New Math Model: (Minimize Final Time)

Subject to differential constraints, the new math model optimal control problem seeks to minimize the final time. Many areas, such as manufacturing, transportation, and energy systems, are concerned with reducing final time to the absolute minimum. It is the desired end-state that must be met in each case, and the optimizer's goal is to achieve those conditions in the shortest amount of time possible.

$$Min_{u(t)} \quad : \quad t_f$$

$$\text{Subject to} \quad : \quad \frac{dx_1}{dt} = u$$

$$\frac{dx_2}{dt} = \cos(x_1^2\,(t))$$

$$\frac{dx_3}{dt} = \sin(x_1^2\,(t))$$

$$x(0) = [\pi/2, 4, 0]$$

$$x_2(t_f) = 0$$

$$x_3(t_f) = 0$$

-

2≤ u(t) ≤ 2

To solve the new math model problem, one must specify a time horizon in the range of 0.0 to 1.0, along with the additional tf parameter, which scales the final time to one. The conditions x2(tf)= 0 and x3(tf)= 0 are binding at the end of the system and prevent it from having a lower final time than the conditions.

Equivalent Form for GEKKO

$$Min_{u(t)(t_f)} \quad : \quad t_f$$

$$\textbf{Subject to} \quad : \quad \frac{dx_1}{dt} = t_f u$$

$$\frac{dx_2}{dt} = t_f \cos(x_1^2\,(t))$$

$$\frac{dx_3}{dt} = t_f \sin(x_1^2\,(t))$$

$$x(0) = [\pi/2, 4, 0]$$

$$x_2(t_f) = 0$$

$$x_3(t_f) = 0$$

$$-2 \le u(t) \le 2$$

2. Basic principles

2.1 Dynamic Optimization

Calculating future outcomes using differential and algebraic equation mathematical models in order to formulate smart policies on the basis of these predictions, dynamic optimization is a decision-making process that can be used to formulate smart policies. This type of analysis can be carried out using a wide variety of tools and techniques that are available. The applications listed below are representative of their kind, as they are both simple in nature and quick to compute. The purpose of this compilation is to demonstrate how to set up, solve, and analyze the problems presented in this collection of problems. In order to discuss alternative strategies, please use the comment sections located at the bottom of each page. While studying the optimal time path for a particular function, dynamic optimization is frequently concerned with the stock-flow relationships that exist between variables at different points in time. [10,11] Certain variables involved are stock concepts, also known as state variables in dynamic optimization, whereas flow concepts are more commonly referred to as control variables in the field of dynamic optimization. For example, in the context of production theory, stocks change from one period to another, and their increase is dependent on both the stocks and the flows that occur during that period. Optimization over time can be expressed as the sum, difference, or product of functions that are also changing over time, with the objective function being the sum, difference, or product of these changing functions.

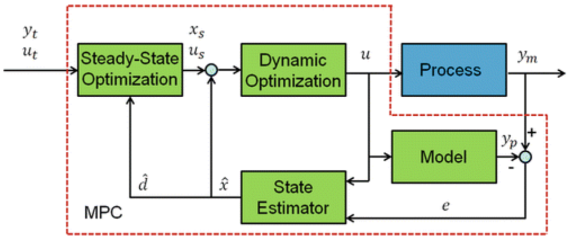


Figure (1) model- predictive control

2.2 Formulation Strategies

In order to achieve efficient and reliable solution of dynamic systems, model formulation is one of the most important factors to consider. Model formulation changes are not intended to alter the equations themselves; rather, they are intended to put them in a form that allows solvers to more easily find an accurate solution. Some of the most important strategies for model creation and formulation are discussed in detail in each of the sections below. Starting with a fundamental introduction to the APMonitor Modeling Language, the discussion moves on to more in-depth topics. Models are divided into sections, which include constants, parameters, variables, intermediates, equations, objects, and connections, amongst other elements. It is necessary for all expressions to be created in one of the previous sections before they can be used in the equations section. Individual parameters or variables are initialized in the order in which they are listed in the model file, with the first parameter or variable being set first. The equations, on the other hand, can be listed in any order because they are all solved at the same time.

2.3 Numerical Differential Equation

Nominally, a numerical method for ordinary differential equations is a method for determining numerical approximations to the solutions of ordinary differential equations (ODEs). In addition, their application is referred to as "numerical integration," although this term can refer to the computation of integrals as well. [6,7,15,16,17] Because of the nature of differential equations, many of them cannot be solved using symbolic computation ("analysis"). If the solution is needed for practical purposes – such as in engineering – a numeric approximation to the solution is frequently sufficient. The algorithms that have been studied here can be used to compute an approximation of this kind. An alternative method is to employ calculus-based techniques to obtain a series expansion of the solution to the problem. There are many different scientific disciplines where ordinary differential equations can be found, including physics, chemistry, biology and economics. On top of all that, some numerical partial differential equation methods convert the partial differential equation into an ordinary differential equation, which has to be solved afterward.

3. Optimal Control

Control is concerned with a number of different aspects. Stability, precision, and speed are all important. A cost that can be computed at each time step and that reflects the quantity we want to minimize is defined as an optimal control cost in optimal control. It could be a matter of time or precision, or both at the same time, or it could be a matter of other variables to optimize (consumption of fuel, etc..). [8,9,20,21] First and foremost, let us consider that the quantity that we are attempting to optimize is that of time. It is the goal of this problem to find the sequence of inputs to the system that will steer it to the desired state in the shortest amount of time possible, given an initial system state and a desired state to reach. This means that once we have identified the optimal controls, there will be no other sequence of controls that will allow us to achieve the desired state in a shorter amount of time. Consider the case of a car, which can be driven in a straight line by varying the amount of pressure applied to the gas pedal. [12,13] The optimal control problem can be expressed as follows: given a starting point of A and a destination of B, how hard should the driver press the gas pedal to bring the vehicle to a stop at B as quickly as possible? Alternatively, in control terms, the problem can be stated as "find the time-optimal sequence of controls u(t) (the angle of the gas pedal) from A to B" (resulting in a motion of the car x(t). In light of our prior knowledge of optimality, we would almost want to formalize this into a mathematical formulation. Something along the lines of:

$$\boldsymbol{min_{x,u} \int_A^B tdt}$$

If you're still with me, you're probably wondering how we're supposed to figure out u(t) and x(t) with this because it's not even in the integral to begin with. [14] This is where the dynamics come into play. In optimal control, all of the systems taken into consideration are dynamic systems, which means that they evolve according to a law of evolution through time of the form x'(t) = f(x(t)).

Take, for example, an air conditioner, where the temperature is governed by the laws of thermodynamics and a missile, which is governed by the laws of motion, as is the case with our hypothetical car. Don't you believe that these factors should be taken into consideration when determining how quickly we can get from point A to point B? There's no need to complicate things; simply include it as a constraint of the minimization problem:

$$\boldsymbol{min_{x,u} \int_A^B tdt}$$

w.r.t

x'(t) = f(x(t))

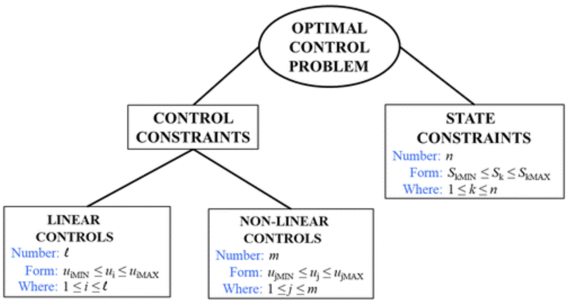


Figure (2) *Optimal Control*

4. GEKKO (Python)

GEKKO Python is intended for large-scale optimization, and it provides access to solvers for problems that are constrained, unconstrained, continuous, and discrete in nature. [2,4,18,19] It is possible to solve problems in linear programming, quadratic programming, integer programming, nonlinear optimization, systems of dynamic nonlinear equations, and multi-objective optimization, among other areas of mathematics. The platform has the ability to find optimal solutions, perform tradeoff analyses, balance multiple design alternatives, and incorporate optimization methods into external modeling and analysis software, among other capabilities. Under the terms of the MIT license, it is available for free for both academic and commercial purposes.

5. Application Modes

This function is used in the minimum and optimal control of the time of arrival of cars, planes, mushrooms, and space vehicles to their destinations, as well as in the reduction of the launch time of missiles, the reduction of the speed of chemical and physical reactions, as well as the reduction of the time of temperature rise in nuclear reactions, among other life-sustaining operations.

6. Numerical Results in new math model

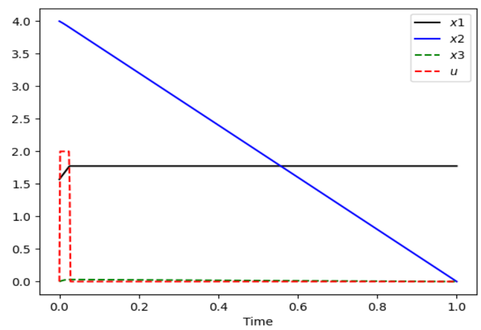
Applying the new approach from the previous objective function, we get the optimal time to solve the problem, as shown in Figure (3,4) and Table (1,2):

	scaled	unscaled
Objective	4.0069592942508189e+02	2.0034796471254094e+03
Dual infeasibility	1.8157819579791169e-10	9.0789097898955843e-10
Constraint violation	3.4237856993968308e-10	3.4237856993968308e-10
Complementarity	7.1364389221260054e-11	3.5682194610630024e-10
Overall NLP error	3.4237856993968308e-10	9.0789097898955843e-10

Table (1)

Solution time : 1.46450000000186 sec

Objective : 2003.47964712541



Figure(3)

We will change the restrictions for the same problem to obtain more satisfactory results in less time than before ($\frac{dx_2}{dt} = t_f \cos(x_1^3(t))$, $\frac{dx_3}{dt} = t_f \sin(x_1^2(t))$).

	scaled	unscaled
Objective	4.0035067119502833e+02	2.0017533559751416e+03
Dual infeasibility	4.1484881698752255e-10	2.0742440849376128e-09

Constraint violation	3.2241764813534246e-10	3.2241764813534246e-10
Complementarity	8.2742470009101295e-11	4.1371235004550645e-10
Overall NLP error	4.1484881698752255e-10	2.0742440849376128e-09

Table(2)

Solution time : 1.05359999999928 sec

Objective : 2001.75335597514

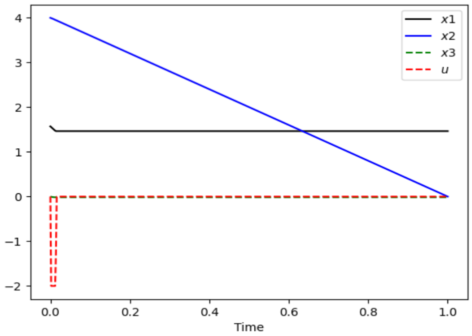


Figure (4)

6.1 The Description

When comparing the results in the first and second tables, we notice that the time and the value of the objective function in the second is better than the first, but the error rate is greater than in the first.

As for the third and fourth figures, we notice that x_2 and x_3 go to the one, while the x_1 is in the form of a straight line between zero and one, then decreases in the third table and rises in the fourth table from its beginning

Conclusion

For the purposes of this research, we developed a new mathematical model for any time-dependent function subject to certain restrictions in order to solve some contemporary problems in various sciences, by controlling the minimum and optimal time of completion, solving the problem using a new algorithm and code in the Python programming language, and we discovered that better results can be obtained. This function is used when the restrictions change in each case or when we have a problem that we will have to deal with in the future.

References

[1] Approach to optimal control problems for differential-algebraic systems based on efficient implicit runge-kutta integration, Applied Mathematical Modelling, 58 (2018), 313{330.

[2] Beal, L., GEKKO for Dynamic Optimization, 2018.

[3] C. Jiang, K. Xie, C. Yu, M. Yu, H. Wang, Y. He and K. L. Teo, A sequential computational

[4] Hedengren, J. D. and Asgharzadeh Shishavan, R., Powell, K.M., and Edgar, T.F., Nonlinear Modeling, Estimation and Predictive Control in APMonitor, Computers and Chemical Engineering, Volume 70, pg. 133–148, 2014.

[5] J. Li, K. Sun and X. Chen, Shortcut to adiabatic control of soliton matter waves by tunable

[6] L. Kong, C. Yu, K. L. Teo and C. Yang, Robust real-time optimization for blending operation of alumina production, Journal of Industrial and Management Optimization, 13 (2017), 1149-1167.

[7] L. Li, C. Yu, N. Zhang, Y. Bai and Z. Gao, A time-scaling technique for time-delay switched systems, Discrete and Continuous Dynamical Systems-S, 13 (2020), 1825-1843.

[8] L. Wang, J. Yuan, C. Wu and X. Wang, Practical algorithm for stochastic optimal control problem about microbial fermentation in batch culture, Optimization Letters, 13 (2019), 527-541.

[9] P. Mu, L. Wang and C. Liu, A control parameterization method to solve the fractional-order optimal control problem, Journal of Optimization Theory and Applications, 187 (2020), 234-247.

[10] Pontes, K.V.; Wolf, I.J.; Embiruçu, M.; Marquardt, W. Dynamic Real-Time Optimization of Industrial Polymerization Processes with Fast Dynamics. Ind. Eng. Chem. Res. 2015, 54, 11881–11893.

[11] Powell, K.M.; Cole, W.J.; Ekarika, U.F.; Edgar, T.F. Dynamic optimization of a campus cooling system with thermal

storage. In Proceedings of the 2013 European Control Conference (ECC), Zurich, Switzerland, 17–19 July 2013; pp. 4077–4082.

[12] T.-Y. Huang, B. A. Malomed and X. Chen, Shortcuts to adiabaticity for an interacting bose{einstein condensate via exact solutions of the generalized ermakov equation, Chaos: An Interdisciplinary Journal of Nonlinear Science, 30 (2020), 053131, 10 pp.

[13] Touretzky, C.R.; Baldea, M. Integrating scheduling and control for economic MPC of buildings with energy storage. J. Process Control 2014, 24, 1292–1300.

[14] Z. Gong, C. Liu and Y. Wang, Optimal control of switched systems with multiple time-delays and a cost on changing control, Journal of Industrial and Management Optimization, 14 (2018), 183{198.

[15] Al-Jilawi, A. S., & Abd Alsharify, F. H. (2022). Review of Mathematical Modelling Techniques with Applications in Biosciences. Iraqi Journal For Computer Science and Mathematics, 3(1), 135-144.

[16] Alridha, A., Wahbi, F. A., & Kadhim, M. K. (2021). Training analysis of optimization models in machine learning. International Journal of Nonlinear Analysis and Applications, 12(2), 1453-1461.

[17] Kadhim, M. K., Wahbi, F. A., & Hasan Alridha, A. (2022). Mathematical optimization modeling for estimating the incidence of clinical diseases. International Journal of Nonlinear Analysis and Applications, 13(1), 185-195.

[18] Alridha, A., Salman, A. M., & Al-Jilawi, A. S. (2021, March). The Applications of NP-hardness optimizations problem. In Journal of Physics: Conference Series (Vol. 1818, No. 1, p. 012179). IOP Publishing.

[19] Salman, A. M., Alridha, A., & Hussain, A. H. (2021, March). Some Topics on Convex Optimization. In Journal of Physics: Conference Series (Vol. 1818, No. 1, p. 012171). IOP Publishing.

[20] Alridha, A., & Al-Jilawi, A. S. (2021, March). Mathematical Programming Computational for Solving NP-Hardness Problem. In Journal of Physics: Conference Series (Vol. 1818, No. 1, p. 012137). IOP Publishing.

[21] Alridha, A. H., & Al-Jilawi, A. S. (2022). Solving NP-hard problem using a new relaxation of approximate methods. International Journal of Health Sciences, 6(S3), 523–536. https://doi.org/10.53730/ijhs.v6nS3.5375