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NEAR SUBTRACTION SEMIGROUPS IN WHICH PRIME IDEALS ARE MAXIMAL-I

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Abstract

In this paper, we will now develop the content of quasi-commutative near_subtraction semi groups, Semi prime ideals, Proper prime ideals, globally idempotent principal ideals. Relation between proper prime ideals and semi prime ideals.

Keywords: The Ideal, “A-divisor”, “A-Potent, Rees quotient”, “near subtraction semigroup”.

1. INTRODUCTION

In the year 1967 Abbott invented the concept of subtraction algebra. After that schein was developed the concept of subtraction semi group in the year 1992 by using the notion of subtraction algebra. Later on near subtraction algebra was developed by Deena in the year of 2007.after that the ideals in near subtraction algebra and some of its properties were studied by Jun et al. This research article actually elaborate the concepts to study about pseudo integral near subtraction semi groups.

2. Preliminaries:

For the preliminaries and definitions refer [1],[2],[3]&[4]

3. Main Content:

THEOREM 3.1 : Let X be near subtraction semigroup with the identity element. If $X \neq 0$ (assume this X has zero) has proper prime ideals in X which is to be maximal, then X is a primary near subtraction semigroup.

Proof : Since X contains identity, then X has itself contains unique maximal ideal M, which is to be the union of all proper ideals in X. If A is to be a (nonzero) proper ideal in X, then $\sqrt{A} = M$ and hence by known result, therefore A is a primary ideal. If T has zero and if $< 0 >$ is a prime ideal, then $< 0 >$ is primary and hence X is primary. If not $< 0 >$ is a prime ideal, then $\sqrt{< 0 >} = M$ and hence by the known result, $< 0 >$ is a primary ideal. Therefore X is a primary ternary semigroup.

NOTE 3.2 : If the near subtraction semigroup X has no identity, then from example, we remark that theorem 3.1, is not true even if the ternary semigroup has a unique maximal ideal. The converse of the theorem 3.1,” is not true even if the semigroup is commutative”.

EXAMPLE 3.3 : Let X = {a, b, 1} be the ternary semigroup under the multiplication given in the following table.

-	*x*	*y*	1
x	*x*	*x*	*ax*
y	*y*	*x*	*x*
1	1	1	*a*

·	a	b	1
a	a	a	a
b	a	b	b
1	a	b	1

Now X is a "primary near subtraction semigroup" in which the prime ideal < a > is not a maximal ideal.

THEOREM 3.4 : Let X be a right cancellative quasi commutative near_subtraction semigroup. If X is a primary "near_subtraction semigroup" or a near subtraction semigroup in which semiprimary ideals are primary, then for any primary ideal Q, √Q is non maximal implies Q = √Q is prime.

Proof : Since √Q is non maximal, then ∃ an ideal A in X so that √Q ⊂ A ⊂ X.

Let a ∈ A\√Q and b, c ∈ √Q. Now Q ⊆ Q ∪ < ab > ⊆ √Q. This implies by theorem 2.3.11, √Q ⊆ √(Q ∪ < ab >) ⊆ √(√Q) = √Q. Hence √(Q ∪ < ab >) = √Q. Thus by hypothesis

Q ∪ < ab > is a primary ideal. Let s ∈ X\A. Then for some natural number n,

asb = snabtc = snab ∈ Q ∪ < ab > . Since a ∉ √Q = √(Q ∪ < ab >) and Q ∪ < ab > is a primary ideal, sb ∈ Q ∪ < ab >. If sb ∈ < ab > then sb = rab for some r ∈ X1 and hence by right cancellative property, we have s = ra ∈ A, a contradiction. Thus sb ∈ Q, which implies, since "s ∉ Q, b ∈ Q" and hence "√Q = Q". Therefore "Q = √Q" and so Q is prime.

THEOREM 3.5 : Let X be a right cancellative quasi commutative "near_subtraction semigroup". If X is either a primary "near_subtraction semigroup" of a near subtraction semigroup in which semiprimary ideals are primary, then "proper prime ideals in X are maximal ideal".

Proof : First we show that if "P is a minimal prime ideal" containing a principal ideal < d >, then "P is a maximal ideal". Suppose P is not a "maximal ideal".

Write M = X\P and A = {x ∈ X : xm ∈ < d > for some m ∈ M}.

Let x, y ∈ A. x, y ∈ A ⇒ xm, ym ∈ < d >, < d > is a principle ideal

⇒ xm – ym = (x – y)m ∈ < d > ⇒ (x – y) ∈ A and hence A is a sub algebra of X.

Let x ∈ A, t ∈ X. x ∈ A ⇒ xm ∈ < d > ⇒ xm = t1d for some t1 ∈ X.

Now txm = t(xm) = t(t1d) = (tt1)d ∈ < d > ⇒ tx ∈ A.

Similarly xt ∈ A and xt ∈ A. Therefor A is an ideal of X.

If x ∈ A, then xm ∈ < d > ⊆ P . " Since P is prime ideal" and hence "x ∈ P". So A ⊆ P.

Let b ∈ P and suppose N = {bkm : m ∈ M and k is a nonnegative integer}.

If bkm, bsm ∈ N for m ∈ M and k, s are nonnegative integers.

Then (bkm - bsm) = (bk – bs)m ∈ N and (bkm)(bsm) = bk+rm2 ∈ N.

Therefore N is a near subtraction subsemigroup of X containing M properly.

If b ∈ P ⇒ bm ∈ P ⇒ bm ∉ M and hence bm ∈ N and bm ∉ M.

Since P is a minimal prime ideal containing < d >, M is a maximal near subtraction subsemigroup not meeting < d >. Since N contains M properly, we have N ∩ < d > ≠ ∅.

So there exist a natural number k such that bkm ∈ < d > ⇒ bk ∈ A ⇒ b ∈ √A.

Since P is prime, by known theorem, P is semi prime and by the known result, P = √P.

So that P ⊆ √A ⇒ P ⊆ √A ⊆ √P = P. therefore P = √A.

By statement of of our theorem A is a "primary" ideal. Since "P is not a maximal ideal",

we have by the known result √A = A ⇒ P = A. Since < d > ⊆ P and < d2 > ⊆ < d >.

Therefore < d2 > ⊆ P and so P is also a minimal "prime ideal" containing < d2 >.

Let B = { y ∈ X : ym ∈ < d2 > for some m ∈ M}. As before, we have B = P.

Since d ∈ P = A = B, so that dm = sd2 for any s ∈ X1.

Since X is a quasi commutative near subtraction semigroup, dm = mnd for some natural number n. By right cancellative property mn = sd, which contradicts the fact that

So that P is maximal. Now if P is any "proper prime ideal", then for any d ∈ P,

< d > is contained in a minimal prime ideal, which is maximal by the above and so that "P is a maximal ideal".

COROLLARY 3.6 : Suppose that T is a (left,lateral,right) cancellative commutative near subtraction semigroup so that either X is a primary near subtraction semigroup or in X an ideal A is primary iff "√A is a prime ideal", then "the proper prime ideals in X are maximal ideal".

Proof : The proof of this corollary is a direct as same as above theorem 3.5

THEOREM 3.7 : Let X be a right cancellative quasi commutative near subtraction semigroup with identity. Then the following are equivalent.

1) "Proper prime ideals in X" which is to be a maximal.

2) X is a primary near subtraction semigroup.

3) Semiprimary ideals in X are primary.

4) If x and y are not units in X, then there exists natural numbers n and m such that

xn = ys and ym = xr for some s, r ∈ X.

Proof : Combining theorem 3.1, and 3.5, we have (1), (2) and (3) are equivalent.

1) ⇒ 4) : Assume (1). Since X contains identity element then ,

T has a unique maximal ideal M,

which is to be only prime ideal in X. If a and b are not units.

If < a > ⊈ M then < a > = X ⇒ 1 ∈ < a > ⇒ a is a unit, which contradicts the hypothesis and hence a ∈ M, similarly b ∈ M. Therefore √< a > = √< b > = M

⇒ b∈ √< a > and a ∈ √< b > ⇒ an = bs, bm = ar for some s, r ∈ X.

4) ⇒ 2) : Let s assume that A be any ideal of T and ab ∈ A. Suppose that a,b are not units in X.

Let b ∉ A, then an = bs ⇒ an+1 = abs ∈ A. Therefore a ∈ √A.

Hence A is to be a left primary. Which is also to similar A is right primary.

Therefore X is primary near subtraction semigroup.

NOTE 3.8 : If T has 0, then the theorem 3.7, is true by assuming nonzero proper prime ideals are maximal.

THEOREM 3.9 : Let X be a right cancellative quasi commutative near subtraction semigroup not containing identity. Then the below statements are equivalent.

1) "X is a primary near subtraction semigroup"

2) Semiprimary ideals in X are primary

3) X has no proper prime ideals.

4) If x, y ∈ T, then there exists a positive numbers which n,m∈N such that xn = ys, ym = xr for some s, r ∈ X.

Proof : (1) ⇒ (2) Since X is primary near subtraction semigroup, then its every ideal is primary. Therefore semiprimary ideal is also primary.

(2) ⇒ (3) : Assume (2). By theorem 3.1.5, "proper prime ideals of X are maximal" and hence "if P is any prime ideal, then P is maximal". Let a, b ∈ X\P. Suppose a – b, ab ∉ X\P ⇒a – b, ab ∈ P ⇒ either a ∈ P or b ∈ P, a contradiction. Therefore a – b, ab ∈ X\P. Clearly X\P satisfies all the properties of near subtraction semigroup. Therefore T\P is near subtraction semigroup. Let a, b ∈ X\P. Then aX ⊈ P and hence P ∪ aX = X ⇒ b ∈ aX

$\Rightarrow$ b = ax for some x $\in$ X. If x $\in$ P, then b $\in$ P, a contradiction. Therefore ax = b has a solution in X\P. Similarly ya = b has a solution in X\P and hence X\P is a near subtraction group. Let us assume that e be the identity element of the near subtraction group X\P. Now e is an idempotent in X and since X is a right cancellative near subtraction semigroup, then e is a left identity of T. Since X is a "quasi commutative near subtraction semigroup:, idempotents in X are commute and hence e is the identity of X, a contradiction, since X has no identity. Therefore X has no proper prime ideals.

3) $\Rightarrow$ 4) : Suppose X does not have any proper prime ideals. So that for any ideal A of X, $\sqrt{A}$=X. Let a,b $\in$ T. Now $\sqrt{< a >} = \sqrt{< b >}$ = X $\Rightarrow$ b $\in \sqrt{< a >}$,

a $\in \sqrt{< b >}$ $\Rightarrow$ bm $\in$ < a >, an $\in$ < b > for some odd natural numbers n, m $\Rightarrow$ xn = ys, ym = xr for some s, r $\in$ X.

4) $\Rightarrow$ 1) : Let us assume tat A be any ideal of X. Let xy $\in$ A, Suppose that x, y are not units in X, then xn = ys $\Rightarrow$ xn+1 = xys $\in$ A $\Rightarrow$ x $\in \sqrt{A}$. Therefore A is left primary. Since X is quasi commutative near subtraction semigroup and right primary. Therefore A is primary and hence X is a primary near subtraction semigroup.

THEOREM 3.10 : Let X be a right cancellative quasi commutative near subtraction semigroup. Then the below conditions are equivalent.

1) "X is to be a primary near subtraction semigroup".

2) Semiprimary ideals in X are primary.

3) Proper prime ideals in X are maximal.

Proof : (1) $\Rightarrow$ (2) : If X has identity. By theorem 3.8, 1) $\Rightarrow$ 2). If X has no identity. By theorem 3.9, 1) $\Rightarrow$ 2).

(2) $\Rightarrow$ (3) : By theorem 3.5, we have 2) $\Rightarrow$ 3).

(3) $\Rightarrow$ (1) : If X has identity. By theorem 3.8 1) $\Rightarrow$ 2). If X has no identity, then X has no proper ideals. By theorem 3.9, 3) $\Rightarrow$ 1)

COROLLARY 3.11 : Let X be a cancellative commutative near subtraction semigroup. Then X is a primary near subtraction semigroup if and only if proper prime ideals in X are maximal. Furthermore X has no idempotents except identity, if it exists.

DEFINITION 3.12 : A near subtraction semigroup X with zero is 0-simple if X2 $\neq$ 0 and X has no nonzero proper ideals.

THEOREM 3.13 : Let X be a semipseudo symmetric "near_subtraction semigroup" with the identity element . so that the following conditions which are to be equivalent.

1) "Proper prime ideals in X which are to be maximal.

2) X is either a simple near subtraction semigroup and so Archimedean near subtraction semigroup or X has a unique prime ideal P so that X is a 0-simple extension of the Archimedean near subtraction subsemigroup P.

"In either case X is a primary near subtraction semigroup" and "X has at most one" globally idempotent principal ideal.

Proof : (1) $\Rightarrow$ (2) : Let us assume that a "proper prime ideals in X are to be maximal". If "X is a simple near subtraction semigroup", then it is abviously X is an Archimedean near subtraction semigroup.

If X is not a simple near subtraction semigroup, then "X has a unique maximal ideal P", which is also the only unique prime ideal. Since "P is a maximal ideal in X",wehave T/P=T\P∪{P}is a

0-simple near subtraction semigroup. Let x, y $\in$ P. Since P is the prime ideal, then its intersection is also prime and hence $\sqrt{< x >} = \sqrt{< y >}$ = P. Since x $\in \sqrt{< x >} = \sqrt{< y >}$

$\Rightarrow$ x $\in \sqrt{< y >}$ $\Rightarrow$ xn $\in \sqrt{< y >}$ $\Rightarrow$ < x >n $\subseteq$ < y > for some natural number n.

This implies xn+2 $\in$ < y >. So P is an Archimedean near subtraction subsemigroup of X.

(2) $\Rightarrow$ (1) : Assume 2), Case-1 : Suppose X is simple. Therefore X does not have any proper

prime ideals and hence there does not exist any proper ideal of X containing P ⇒ P is maximal. Therefore 1) is true.

Case-2 : Suppose X is not simple. Then X "has unique proper prime ideal P such that X is a 0-simple extension of P". Therefore X/P is 0-simple. By the known theorem, "X is a primary near subtraction semigroup". Let us assume that < x > and < y > be two globally proper idempotent principal ideals. Then √< x > = √< y > = P. So by the known theorem

< x >n = < y > for some natural number n. Since < x > is globally idempotent, <x> ⊆ <y>. Similarly we can show that <y> ⊆ < x >. Therefore < x> = < y > and also < x > = < y >. So that X has at most two globally proper idempotent primary ideal.

COROLLARY 3.14 : Assume that X is to be a "duo_near subtraction semigroup" with identity. Then the below conditions are equivalent.

1) the Proper prime ideals in X which are to be a maximal.

2) X is either a near subtraction group and so Archimedean or X has a unique prime ideal P so that X = G ∪ P, here G stands for near subtraction group of units in X and P is an Archimedean near subtraction subsemigroup of X.

In either case "X is a primary near subtraction semigroup" and "X has at most one idempotent different from identity".

Proof : Assume that "X is a duo near subtraction semigroup" which is not a "near subtraction group, then X has a unique maximal ideal M" and hence M is the only unique prime ideal, by assuming (1). Now X\M is the near subtraction group of units in X. By theorem 3.13, (1) and (2) are equivalent. Clearly "X is a primary near subtraction semigroup". If e and f are two proper idempotents in X, then < e > and < f > are two globally idempotent principal ideals. By theorem 3.13, e = f. hence the theorem.

COROLLARY 3.15 : Assume that X is to be a commutative near subtraction semigroup with identity. Then the below conditions are to be equivalent

1) the Prime ideals which are to be maximal.

2) X is either a near subtraction group and so archimedian or X has a unique prime ideal P such that X = G ∪ P, here G is the near subtraction group of units in X and P is an Archimedean near subtraction subsemigroup of X.

In either case X is to be a primary near subtraction semigroup and X has at most one idempotent which is to be different from identity.

Proof : the proof is same as above Corollary 3.14.

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