

# Evaluation of Hydraulic Optimality using optimality indicator in Water Distribution Networks

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**Abstract** Hydraulic optimality discusses the concept of water circulation, and balance between pipe diameters and flow required in each junction to avoid redundancy, and achieving shortest path from source to consumers. Optimality of water distribution systems is generally overlooked in design. In this study an improved linear model is developed, to evaluate hydraulic optimality of water distribution networks through a proposed hydraulic optimality indicator ranging from 0 to 1. A case study of simplified network is considered to illustrate the application of proposed methodology. Through the application of this methodology, hydraulic optimality indicator can be improved from 89% to 95%, and junctions ranking similarity can be raised from 9.5% to 28.5% by upgrading the network through changing diameters of critical links located on the critical path, and this leads to energy saving, increasing water quality, and achieving sustainable water distribution network.

**Keywords** Water Networks, Criticality Analysis, Design Optimality, Hydraulic Performance Indicator, Linear Optimization Model.

## 1. Introduction

Water distribution network is the most important part of water supply systems with capital cost of exceeding 60 % of the total cost of the system. The energy consumed in pumping water in a distribution network may also exceed 60 % of the total energy consumption of the system [1]. Design of water distribution network follows design criteria such as velocity, hydraulic gradient slope, and minimum pressure. If the design follows the ranges of these design criteria, it does not mean it achieves hydraulic optimality. This issue has led to an increase in the modeling of WDN (water distribution network) and evaluating of water hydraulic optimality in drinking water distribution systems and the concept of performance indicators is appeared. When the hydraulic optimality achieved in water distribution network, the runtime of water in network decreased because the water takes the shortest path

from source to consumers, the cost of energy is decreased, and finally the hydraulic optimality affects the retention time of water in network and the quality of water delivered to consumers. Water distribution systems are generally designed with methodologies based on trial-and-error basis, which generates feasible results. However, these trials are not the most economical and optimal solution since they do not consider the hydraulic optimality of the network [2].

(caballero, et al, 2019) defined the optimal design of water distribution network as minimizing the total cost of water distribution network, which depends mainly on pipe diameters and flow direction of water in pipes, and taking in consideration the uncertainty of demand at each junction [3]. (Salcedo.R, 2020) developed stochastic mathematical model to determine the configuration of the WDN that minimizes the total annualized cost (TAC) under nodal water demand uncertainty [4].

(Garcia, 2018) defined the hydraulic optimality as

the solution of problems, such as infrastructure deterioration, that cause losses due to leaks, decreasing of water capacity transported to consumers, failures in system components (pumps, valves, pipes, etc.); also, increased maintenance and operating costs, poor fluid quality due to constant service interruptions, and decreased reliability of the system, generating problems to meet the required demand and pressure. In recent years researches related to water distribution networks has the priority to optimize the design of a DWDS, to ensure the development and proper functioning of a society [5]. (Saldarriaga et al, 2010) have raised a methodology of optimization by finding optimal hydraulic solution with minimum cost [6]. (Bonilla-Granados et al, 2021) defined the concept of the resilience of a WDS as the ability of a network to overcome a failure with minimum cost analysis and resilience index is used to know if the network is optimum or not [7]. (Gomes, 2009) developed a mathematical model called Lenhsnet, which aims to optimize pipe sizing, achieve hydraulic optimality, and minimize the capital cost of pumping energy[8].(Al-Zahrani et al, 2005) defined hydraulic reliability as the ability of the network to deliver water to consumers in the required quantity and quality at suitable pressure head [9].

(Sarbu et al, 2014) made a mathematical model for optimal path in water distribution network based on Graph-Theory [10]. (Lee, 2019) established anew algorithm for optimization of water distribution

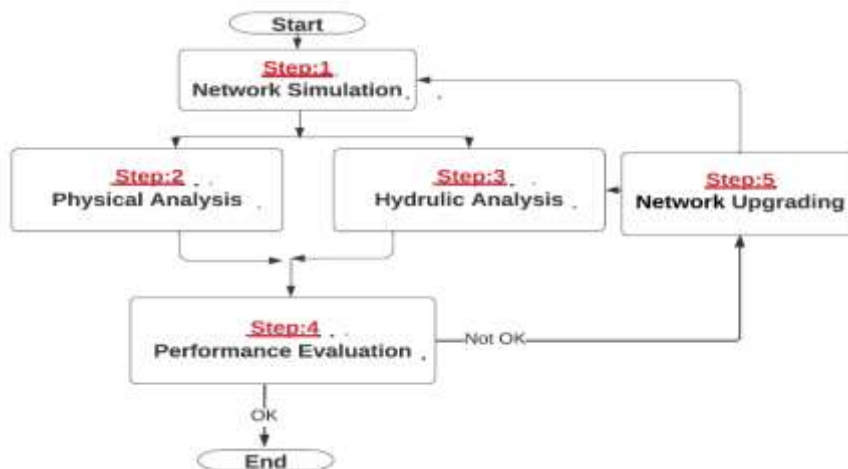
networks, the developed algorithm is Implemented to allow the design of looped system with minimal length ensuring least cost, reliability of the network, and availability of water [11,12]. (Fuso et al., 2020) used the approach of optimality to overcome the lack of consideration of both uncertainty in the model parameters and in the future operating conditions [13, 14].

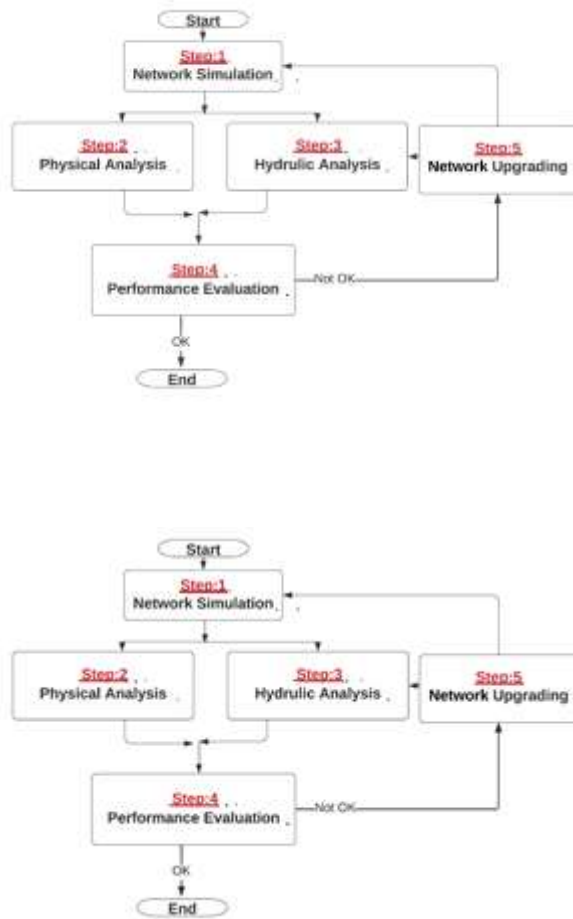
In this paper the hydraulic optimality is defined as delivering water from source to consumers in a shortest path, and it is achieved by balance between pipe diameters and demand required and checked by hydraulic optimality indicator travel distance ratio (TDR), and junction's similarity ranking (JRS). Also this paper develops a linear model to determine the shortest travel distance for each junction and the shortest travel length for the network, junctions are ranked according to the shortest travel distance from physical parameters and ranked again according to hydraulic grade from hydraulic analysis then comparison between two ranks done to check the effect of network upgrading.

## 2. Methodology

The methodology is divided into 5 main steps as described in the flow chart presented in Figure 1.

A detailed calculations procedure for each step are explained in order to illustrate the methodology.





**Figure 1.** Flowchart of the Adopted Methodology for Water Network hydraulic optimality

**2.1. Network Simulation**

This step includes simulation of the water network using input physical data of network such as: pipes, junctions, pumps, and valves data. Output data of this step includes the following data: Model structure for case study to match the design constrains of velocity and pressure.

**2.2. Physical Analysis**

This step includes calculations of physical properties of water distribution network such as shortest travel distance (STD), junctions are ranked according to STD from nearest to farthest.

STD for each junction: It is the shortest distance, which water travels from source to each junction. The travel distance from source to junction should be minimum to avoid redundancy and achieve hydraulic optimality of network. Shortest distance of each junction is calculated by linear programming solver using excel, and physical travel distance of the network

can be calculated as shown in Equation 1

$$\text{Physical Travel Distance of the Network} = \frac{\sum_{j=1}^{j=n} q * L_j}{\sum q} \tag{1}$$

Where:

n: no of junctions.

q: calculated demand at each junction.

L<sub>j</sub>: Shortest Travel Distance from reservoir to each junction.

After determination of the shortest travel distance for ach junction, junctions can be ranked from nearest to farthest, and critical path can be determined to farthest junction.

**2.3. Hydraulic Analysis**

In this step hydraulic analysis of water networks is undertaken using one of approved software like water gems, flow of water in pipes, hydraulic grade of each

junction, and hydraulic gradient slope of each pipe are determined.

Pipes are ranked according to head loss gradient (safe- medium-critical), and junctions are ranked according to hydraulic grade to 3 categories (safe-medium-critical), then the critical junctions and critical pipes are determined. Critical junctions: they are junctions which have the least hydraulic grade in the water distribution network. Critical junctions are the farthest junction from the source of water, or the junction which the water takes along path to reach it due to unbalanced pipe diameters to flow required. Critical pipes are pipes which have the maximum head loss gradient in the network. Maximum head loss gradient due to small diameter so the head loss value is large.

Using flow in each pipe and length of pipe the actual travel distance of the water distribution network can be define as actual distance the water will travel from water source to reach any junction and can be calculated from the relation mentioned in Equation 2.

$$\text{Hydraulic Travel Distance} = \frac{\sum_{P=1}^m Q \cdot L}{\sum Q} \quad (m) \quad (2)$$

Where:

m: no of pipes.

Q: calculated flow at each pipe.

L: Actual length of each pipe.

### 2.4. Performance Evaluation

This step includes comparison between hydraulic analysis results and physical analysis results, and evaluate hydraulic optimality of the network through calculation of hydraulic optimality indicator travel

distance ratio (TDR).

(TDR): The ratio between physical travel distance of water through the network and hydraulic travel distance of the network, it can be calculated from the relationship mentioned in Equation 3, and this value is from 0 to 1

$$\text{Travel Distance Ratio for the network (TDR)} = \frac{\text{Physical Travel Distance}}{\text{Hydraulic Travel Distance}} \quad (3)$$

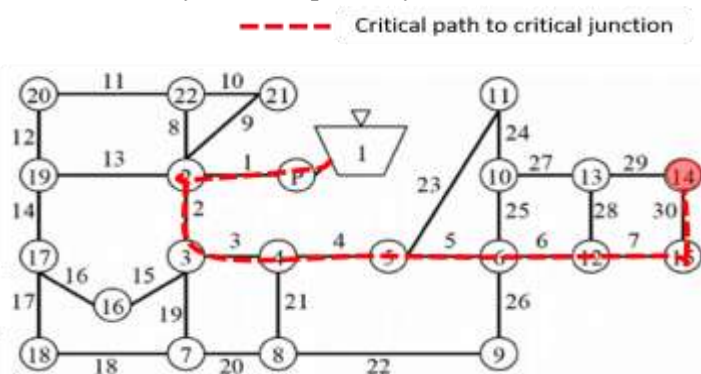
And compare between junctions ranking from physical ranking and hydraulic ranking and calculate the ratio of junctions ranking similarity (JRS).

### 2.5. Network Upgrading

The purpose of this step to upgrade the design of WDN to increase hydraulic optimality through increasing the diameters of critical pipes in the critical path to increase the TDR value

### 3. Case Study

The simplified water network consists of 30 pipes, 22 nodes, and 9 loops, (Kim et al. 1994). The network is fed by a single pump of 4.52 kW from a 71 m constant head reservoir. The H–W coefficient for all pipes in the network is 130. The minimum head limitation for this network is 15m above ground level. The details of the distribution are described below: is illustrated in Figure 2



**Figure 2.** Simplified Water Supply Network [13].**3.1. Network Simulation****Table 1.** Water Network Pipe Data [13].

Pipe Label	Diameter (mm)	Length (m)	Pipe Label	Diameter (mm)	Length (m)
P-1	300	165	P-17	80	72
P-2	200	124	P-18	80	347
P-3	200	118	P-19	100	98
P-4	100	81	P-20	100	118
P-5	100	134	P-21	100	98
P-6	100	136	P-22	100	81
P-7	80	202	P-23	100	236
P-8	80	135	P-24	100	102
P-9	80	176	P-25	80	92
P-10	80	113	P-26	100	100
P-11	80	335	P-27	100	136
P-12	80	135	P-28	80	90
P-13	80	345	P-29	100	201
P-14	80	114	P-30	100	90
P-15	80	193	P-32	200	29
P-16	80	162			

**Table 2.** Water Network Junction Data [13].

Junction Label	Elevation (m)	Demand (m <sup>3</sup> /day)	Junction Label	Elevation (m)	Demand (m <sup>3</sup> /day)
J-2	56.4	153	J-13	59.3	38
J-3	53.8	70	J-14	59.8	63
J-4	54.9	59	J-15	59.2	445
J-5	56	75	J-16	53.6	108
J-6	57	68	J-17	54.8	80
J-7	53.9	63	J-18	55.1	55
J-8	54.5	48	J-19	54.2	119
J-9	57.9	42	J-20	54.5	124
J-10	62.1	30	J-21	62.9	31
J-11	62.8	42	J-22	62.8	31
J-12	58.6	38			

**3.2. Physical Analysis**

Using linear programming solver excel. Calculations of shortest travel distance of each junction is calculated

then the theoretical travel distance of the water distribution network is calculated as showed in Table 3.

**Table 3.** Shortest Travel Length for Junctions and ranking of junctions according to physical analysis

Junction Label	Shortest Travel Distance- STD (m)	Demand (m <sup>3</sup> /day)	Q*STD (m <sup>3</sup> .m/day)	Ranking according to STD
J-2	194	153	29,682	1(Nearest)
J-3	318	70	22,260	2
J-4	436	59	25,724	6
J-5	517	75	38,775	8
J-6	651	68	44,268	12
J-7	416	63	26,208	5
J-8	534	48	25,632	9
J-9	615	42	25,830	11
J-10	743	30	22,290	15
J-11	753	42	31,626	16
J-12	787	38	29,906	18
J-13	877	38	33,326	19
J-14	1,078	63	67,914	21(Farthest)
J-15	989	445	440,105	20
J-16	511	108	55,188	7
J-17	653	80	52,240	13
J-18	763	55	41,965	17
J-19	539	119	64,141	10
J-20	664	124	82,336	14
J-21	370	31	11,470	4
J-22	329	31	10,199	3
Total	1782	Total	1,181,085	

$$\text{Hydraulic Travel Distance of the network} = \frac{1,181,085}{1,782}$$

=662.7 (m).

After determination of critical junction, which is the farthest junction from water source as shown in Figure 2, the critical path can be determined from linear programming Excel solver. The critical path from R-1 to J-14 is shown in Figure 2.

### 3.3. Hydraulic Analysis

The head loss gradient for each pipe is calculated as shown in Table 4 to rank pipes to (safe-medium-critical) as shown in Figure 3, and hydraulic grade for each junction is calculated as shown in Table 5 to rank junctions to (safe-medium-critical) as shown in Figure 3, then actual travel distance of the network can be calculated from results in Table 6.

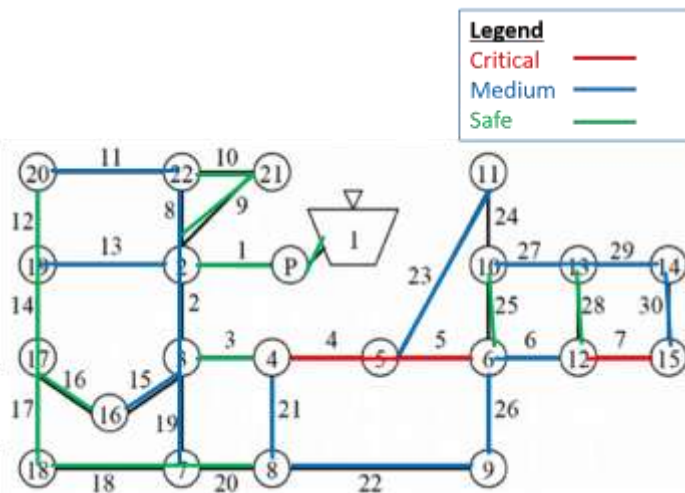
**Table 4.** Head loss gradient of pipes

Pipe Label	Hydraulic Grade slope(s) (m/km)	Pipe Label	Hydraulic Grade slope(s) (m/km)	Pipe Label	Hydraulic Grade slope(s) (m/km)
P-1	0.345	P-13	1.825	P-25	0.571
P-2	1.411	P-14	0.019	P-26	2.167

Pipe Label	Hydraulic Grade slope(s) (m/km)	Pipe Label	Hydraulic Grade slope(s) (m/km)	Pipe Label	Hydraulic Grade slope(s) (m/km)
P-3	0.609	P-15	2.195	P-27	2.48
P-4	7.731	P-16	0.205	P-28	0.376
P-5	5.368	P-17	0.09	P-29	2.813
P-6	2.623	P-18	0.723	P-30	1.84
P-7	3.794	P-19	2.037	P-32	2.489
P-8	1.099	P-20	0.442	P-24	1.847
P-9	0.674	P-21	1.831	P-25	0.571
P-10	0.263	P-22	2.841	P-26	2.167
P-11	1.437	P-23	2.476	P-27	2.48
P-12	0.005	P-24	1.847	P-28	0.376

**Table 5.** Hydraulic Grade at Junctions

Junction Label	Hydraulic grade(m)	Ranking	Junction Label	Hydraulic grade(m)	Ranking
J-2	79.87	1	J-13	77.89	19
J-3	79.7	4	J-14	77.32	20
J-4	79.62	5	J-15	77.16	21
J-5	79	14	J-16	79.27	8
J-6	78.28	16	J-17	79.24	12
J-7	79.5	6	J-18	79.25	9
J-8	79.44	7	J-19	79.24	10
J-9	79.21	13	J-20	79.24	11
J-10	78.23	17	J-21	79.75	2
J-11	78.41	15	J-22	79.72	3
J-12	77.92	18			



**Figure 3.** Ranking of Pipes according their Criticality

**Table 6.** Calculation of Actual Travel Distance

Pipe Label	Q(m3/day)	Length(m)	Q*L	Pipe Label	Q(m3/day)	Length(m)	Q*L
P-1	1,782	165	294,030	P-20	113	118	13,334
P-2	1,312	124	162,688	P-21	244	98	23,912
P-3	833	118	98,294	P-22	309	81	25,029
P-4	531	81	43,011	P-23	287	236	67,732
P-5	436	134	58,424	P-18	82	347	28,454
P-6	296	136	40,256	P-19	258	98	25,284
P-7	201	202	40,602	P-20	113	118	13,334
P-8	103	135	13,905	P-21	244	98	23,912
P-9	79	176	13,904	P-22	309	81	25,029
P-10	48	113	5,424	P-23	287	236	67,732
P-11	119	335	39,865	P-24	245	102	24,990
P-12	5	135	675	P-25	72	92	6,624
P-13	135	345	46,575	P-26	267	100	26,700
P-14	11	114	1,254	P-27	287	136	39,032
P-15	150	193	28,950	P-28	58	90	5,220
P-16	42	162	6,804	P-29	308	201	61,908
P-17	27	72	1,944	P-30	245	90	22,050
P-18	82	347	28,454	P-32	1,782	29	51,678
P-19	258	98	25,284	Total	1,782		1,318,552

Hydraulic Travel Distance of the network =  $\frac{1,318,552}{1,782} = 739.9$  (m)





### 3.4. Performance Evaluation

The purpose of this step to compare between ranking of

junctions according to STD and HG as shown in Table 7, and check if the network design achieves hydraulic optimality or not through TDR.

**Table 7.** Comparison of junctions ranking

Junction Label	Ranking according to STD	Ranking according to HG	Junction Label	Ranking according to STD	Ranking according to HG
J-2	1	1	J-13	19	19
J-3	2	4	J-14	21	20
J-4	6	5	J-15	20	21
J-5	8	14	J-16	7	8
J-6	12	16	J-17	13	12
J-7	5	6	J-18	17	9
J-8	9	7	J-19	10	10
J-9	11	13	J-20	14	11
J-10	15	17	J-21	4	2
J-11	16	15	J-22	3	3
J-12	18	18			

From the previous table, the percentage of similarity between physical ranking and hydraulic ranking of junctions is 9.5%, 2 junctions of 21 junctions only have the same ranking from physical view and hydraulic view.

$$TDR = \frac{\text{Physical Travel Distance}}{\text{Hydraulic Travel Distance}} * 100$$

$$TDR \text{ for the network} = \frac{662.7}{739.9} * 100 = 89.5\%$$

### 3.5. Network Upgrading

In this step, diameters of critical pipes in critical path as shown in Figure 3 in red color are increased, then hydraulic analysis is conducted again and the results are collected and then actual travel distance and travel distance ratio are calculated again to study the effect of upgrading scenario on TDR value, and on the similarity of junctions ranking as shown in Table 7

**Table 8.** Upgrading of Water Distribution Network

Pipe Label	Diameter (mm)	Length (m)	Pipe Label	Diameter (mm)	Length (m)	Pipe Label	Diameter (mm)	Length (m)
P-1	300	165	P-11	80	335	P-21	100	98
P-2	200	124	P-12	80	135	P-22	100	81
P-3	200	118	P-13	80	345	P-23	100	236
P-4	200	81	P-14	80	114	P-24	100	102
P-5	200	134	P-15	80	193	P-25	80	92
P-6	100	136	P-16	80	162	P-26	100	100
P-7	150	202	P-17	80	72	P-27	100	136
P-8	80	135	P-18	80	347	P-28	80	90
P-9	80	176	P-19	100	98	P-29	100	201
P-10	80	113	P-20	100	118	P-30	100	90

Pipe Label	Diameter (mm)	Length (m)	Pipe Label	Diameter (mm)	Length (m)	Pipe Label	Diameter (mm)	Length (m)
						P-32	200	29

**Table 9.** Network Upgrading Results

Pipe Label	Q(m <sup>3</sup> /day)	Length(m)	Q*L	Pipe Label	Q(m <sup>3</sup> /day)	Length(m)	Q*L
P-1	1,782	165	294,030	P-17	36	72	2,592
P-2	1,317	124	163,308	P-18	92	347	31,924
P-3	916	118	1,08,088	P-19	185	98	18,130
P-4	772	81	62,532	P-20	30	118	3,540
P-5	564	134	75,576	P-21	86	98	8,428
P-6	347	136	47,192	P-22	68	81	5,508
P-7	370	202	74,740	P-23	159	236	37,524
P-8	101	135	13,635	P-24	117	102	11,934
P-9	78	176	13,728	P-25	150	92	13,800
P-10	46	113	5,198	P-26	26	100	2,600
P-11	116	335	38,860	P-27	237	136	32,232
P-12	8	135	1,080	P-28	61	90	5,490
P-13	132	345	45,540	P-29	139	201	27,939
P-14	6	114	684	P-30	76	90	6,840
P-15	145	193	27,985	P-32	1,782	29	51,678
P-16	37	162	5,994	Total	1,782	Total	1,238,329

$$\text{Actual Travel Distance of the network} = \frac{1,238,329}{1,782} = 694.9 \text{ (m)}$$

$$\text{TDR for the network} = \frac{662.7}{694.9} * 100 = 95.3\%.$$

**Table 10.** Comparison of junctions ranking after upgrading

Junction Label	Ranking according to STD	Ranking according to HG	Junction Label	Ranking according to STD	Ranking according to HG
J-2	1	1	J-13	19	19
J-3	2	4	J-14	21	20
J-4	6	5	J-15	20	21
J-5	8	14	J-16	7	8
J-6	12	16	J-17	13	12
J-7	5	6	J-18	17	9
J-8	9	7	J-19	10	10
J-9	11	13	J-20	14	11
J-10	15	17	J-21	4	2
J-11	16	15	J-22	3	3
J-12	18	18			



From the previous table, the percentage of similarity between physical ranking and hydraulic ranking of junctions is 28.5%, 7 junctions of 21 junctions have the same ranking from physical view and hydraulic view, so increasing hydraulic optimality means the increasing of similarity percentage of junctions according to STD, and HG.

#### 4. CONCLUSIONS AND RECOMMENDATIONS

Hydraulic optimality analysis of water distribution networks doesn't only mean achieving the design constrains of velocity and pressure in design of WDN, but also mean the water moves in shortest path from source to consumer and decreasing the retention time of water in distribution network due to balance between pipe diameters and flow required to achieve better quality and quantity of water to consumers.

Shortest travel distance for each junction can be determined by linear programming model using excel solver, and the physical travel distance, and hydraulic travel distance of the network can be calculated by the mathematical model developed during this research.

Hydraulic optimality can be evaluated in the water distribution network by hydraulic optimality indicator TDR as shown in the following relationship:

$$TDR = \frac{\text{Physical Travel Distance}}{\text{Hydraulic Travel Distance}} * 100 \quad (3)$$

Hydraulic optimality analysis helps the designer to rank of junctions according to STD, and HG and comparison between them and taking in consideration this comparison to be indicator of hydraulic optimality degree. When the similarity between ranking of STD, and HG increases, the hydraulic optimality degree increases.

Hydraulic optimality analysis helps the designer and decision makers to decide the links, which have a problem or changing the diameters of these links will improve the hydraulic optimality of the network through ranking the pipes according to hydraulic gradient slope and critical links are the links, which have the maximum value of hydraulic gradient slope (S).

The results from this manuscript can be a guide for designer and stakeholders to achieve hydraulic optimality of water distribution network, decreasing

the cost of pumping, achieving energy saving, and help the developing countries in energy problems they face.

Finally, hydraulic optimality check must be applied for each water distribution network before implementation to make sure that water moves in shortest path from source to consumer and decreasing the retention time of water in distribution network to improve water quality in WDN.

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