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Construction Of Mathematics Problem-Based On APOS Theory To Encourage Reflective Abstraction Viewed From Students' Creative Thinking Profile

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Abstract

The highest category within the cognitive process category is 'to create,' which refers to the creative process. The term 'create' is linked to three cognitive processes: generating, planning, and producing. This conforms to the framework of abstraction-reflective knowledge construction mechanisms theory. In order to generate new scientific knowledge, students must engage in reflective abstraction. This study aims to describe the seven processes of reflective abstraction construction based on the student's APOS theory in constructing mathematical problems, namely in the basic introductory mathematics concepts. This study involved sixty students as the research participants/subjects. Everyone was given a test question regarding the introduction of complex numbers. Based on these answers, four students were chosen according to their level of creative thinking: first, second, third, and fourth. An interview was conducted based on the test results for each level of creative thinking. To confirm the validity and reliability of the research, triangulation was accomplished by displaying data from video interview results and comparing them to students' written test results. Students at the fourth level of creative thinking demonstrated seven steps of reflective abstraction construction based on APOS theory, i.e., interiorization, coordination, reversal, encapsulation, de-encapsulation, generalization, and specialization. The findings demonstrated that students at the first level of creative thinking with a high level of thinking ability could perform a specialization process, that is, infer solutions from the roots of quadratic equations that can be solved using a complex number system. Therefore, it can be stated that students can produce accurate descriptions when applying the scheme to a larger collection of phenomena.

Keywords: APOS theory, creative thinking, mathematics problem, reflective abstraction.

INTRODUCTION

Since mathematics is an "abstract science" (Bachtiar & Susanah, 2021), it requires careful study to understand its concepts fully. Learning mathematics can be accomplished to understand the concepts of mathematics fully. Learning mathematics transforms knowledge and action, i.e., cognitive, affective, and psychomotor (Simon, 2004; Rakwi, Shafie & Ali, 2021). This transformation process is carried out from school mathematics to advanced mathematics. Many studies have shown difficulties and tensions in transitioning from school mathematics to advanced mathematics (Yeni, 2015; Foley et al., 2017; Ramirez et al., 2018; Medeiros et al., 2018; González-Martín et al., 2021; Agustyaningrum et al., 2021, Hyland & O'Shea, 2021; Hamukwaya & Haser, 2021; Rusliah et al., 2021).

In mathematics, abstraction is a part that cannot be separated from its essential and fundamental aspects (Kramer, 2007; Hakim et al., 2019). According to Nardi (2017), today's students must demonstrate a strong foundation in mathematical abstraction. In accordance with the viewpoint of Hong and Kim (2016), the most important step in the process of discovering new concepts can be accomplished through the use of abstraction. This involves constructing the differences between various objects viewed from a variety of perspectives in order to identify both similarities and differences between the objects being considered. Abstractions can help teach mathematics and prevent students from developing misunderstandings about the concepts they are taught (Rich & Yadav, 2020; Kadarisma et al., 2020).

According to Beth & Piaget (1966), confirmed by the study conducted by Gray & Tall (2007) and Krnxhi & Gjoci (2017), there are three abstraction theories for explaining students' mathematical and logical structure constructs, i.e., empirical, pseudo-empirical, and reflective. First, empirical abstraction is a process for constructing the meaning of various objects' natures (Dubinsky, 1991). Second, pseudo-empirical abstraction based on Piaget (Dubinsky, 1991; Tall, 2002; Rif et al., 2019) is a building mechanism in which learners find the qualities of objects by picturing objects, the meaning of the attributes of action on objects (concretely converted into abstracts). Third, reflective abstraction is a construction mechanism regarding actions on objects and operations into thematic objects on thought or assimilation concerning the category of mental operations and abstractions to mental objects.

The research, in this case, was focused on constructing reflective abstractions. The theory of reflective abstraction was first introduced by Piaget and then developed by Dubinsky. Reflective abstraction is more robust and broader in constructing a concept than an empirical abstraction (Beth & Piaget, 1966). Moreover, according to Cetin and Dubinsky (2017), the description of the concept on the reflective abstractive is independent of any context and, therefore, can be applied to the concept in all situations in which it arises.

Piaget (in Dubinsky, 2002) proposed the notion of reflective abstraction to characterize an individual's production of logical-mathematical structures during cognitive development. Regarding cognitive features, the most important trait is a reflective abstraction, which helps the learner generate new concepts (Bachtiar & Susanah, 2021). Hence reflective abstraction is the construction of a new concept or a preconceived concept throughout an individual's continuing cognitive development.

According to Goodson-Espy (2005), reflective abstraction activities can reveal how learners create conceptual knowledge. This method is evident when they provide justifications for their decisions. According to constructivist theory, a person's knowledge is the formation/construction of the person himself. Knowledge formation begins if a person builds a scheme of abstraction outcomes that have already been possessed when he gets into problems. Such students' schemes are produced in order to form new knowledge. Thus, the scheme of knowledge at each stage of development and the scheme of the corresponding pattern of behavior are the results of reflective abstraction. Simultaneously, the scheme is a person's mental structure (Suparno, 2001).

Knowledge will be formed if students actively engage in the construction process (Subanji, 2015). Reflective abstraction is important for forming sophisticated mathematical conceptions, according to Nisa, Waluya, and Mariani (2020), since mathematical structures are processed through reflective abstractions. Furthermore, reflective abstraction is used to build knowledge, such as high-level mathematics (Netti, 2020). Developing complex mathematical concepts at the first level of college students is a crucial introduction to mathematics. For example, researchers may assign students to solve mathematical problems involving the roots of quadratic equations. However, in this case, the quadratic equation lacks a solution of the roots of the square in the form of real numbers, whereas pupils at the beginning of the basic introduction to mathematics receive content up to the idea of real numbers. The solution for students to answer the test questions shows them constructing reflective abstractions.

Based on earlier research, the subjects of this study had already been classified as creative thinking. De Bono (in Barak & Doppelt, 2000) identifies four levels of achievement in developing creative thinking skills: thinking awareness, thinking observation, thinking strategies, and thinking reflection. Level 1 is the lowest level of creative thinking because it merely conveys students' awareness of the necessity to fulfil their objectives. Level 2 indicates higher creative thinking since students must illustrate how they observe an implication of their choice, such as employing unique components or algorithms. Level 3 is the next level of creative thinking because students must adopt a strategy and coordinate between the numerous explanations in their assignments. They must decide on the desired level of detail and how to communicate the sequence of events or the logical conditions of the system of operations. Level 4 is the highest level of creative thinking because students must compare the properties of the final output to a set of goals, explain the conclusions to challenges encountered during the development process, and offer suggestions for enhancing the planning and building process. This level of creative thinking ability describes thinking processes in general, not only in mathematics.

Furthermore, the highest category in the category of cognitive processes is 'create,' which deals with the creative process. The term "create" refers to three cognitive processes: generating, planning, and producing. Generating is a divergent phase in which learners are asked to pay attention to potential solutions to a challenge. If they get the probability of completion, a method is chosen as an action plan. Finally, the plan is put into action by building a settlement. As shown in Figure 1, the formation of cognitive structures is owing to reflective abstractions, which is consistent with the abstraction-reflective knowledge construction mechanism framework theory of the APOS Theory (Action, Process, Object, Schema) (Dubinsky, 2002; Meel, 2003). Dubinsky (2002), Meel (2003), Arnawa et al. (2007), Maharaj (2010), Arnon et al. (2014), and Utami et al. (2019) state that there are five constructions in reflective abstraction, i.e., interiorization, coordination, reversal, encapsulation, and generalization, to explain this theory and relate concepts in mathematics.

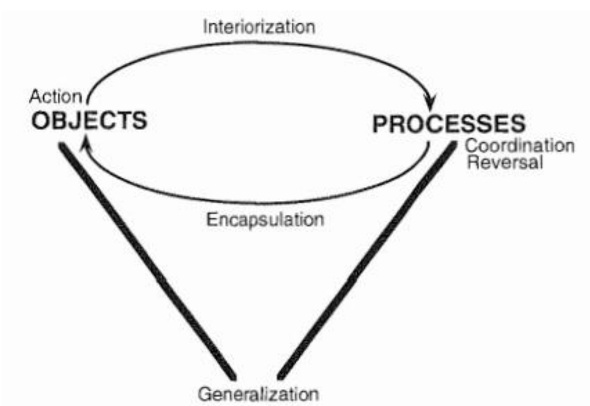

Figure 1. Construction Mechanisms of Reflective Abstraction
(Dubinski, 2002; Meel, 2003; Arnawa et al., 2007; Maharaj, 2010; Arnon et al., 2014)

The researchers aimed to develop a student's thought process about reflective abstraction constructions in understanding the concept of numbers at the roots of quadratic equations because there are students who are met by researchers providing answers based on constructions more than required, namely related to the solution of the quadratic equation system where when the solution does not exist in the real number system, the student can explain in detail related to the system complex numbers. Meanwhile, students at the initial level have not been given complex analysis courses because they should have been given at the upper level in higher education after they got the basic introduction to mathematics courses in this research at the initial level, then number theory, calculus and real analysis (Wahyuni & Kharimah, 2018; Stephani & Yolanda, 2021). To accommodate these students, the researchers intended to develop research conducted by Dubinsky (2002), Meel (2003), Arnawa, Maharaj (2010), Arnon et al. (2014), Cetin & Dubinsky (2017), Utami, et al. (2019), by adding a construction process of reflective abstraction, namely specialization stage after the generalization stage.

As a result, more research is needed to confirm the researchers' assumption that there are students at the specialization stage in the mechanism of reflective abstraction construction in understanding the concept of numbers in solving mathematical problems on the roots of quadratic equations. If this is the case, how does the mechanism of reflective abstraction construction at the specialization stage support understanding the concept of numbers and solving mathematical problems at the roots of quadratic equations?

METHOD

Research Subjects and Instruments

This research employed a qualitative method. The main instrument was the researchers themselves; the additional instruments were test questions and interview guidelines. The test questions are given to the whole subject in the search for the roots of the quadratic equation. The subjects in this study were 60 students. The following is a mapping carried out on students at the level of creative thinking, as shown in Table 1.

Table 1. De Bono's Level of Creative Thinking

Description	**Number of Students**
Level 1: Awareness of Thinking: The students only express the student's awareness of the need to complete the task.	**11**
Level 2: Observation of Thinking: The Students must show how they observe an implication of their choice, such as using unique components or algorithms.	**20**
Level 3: Thinking strategy: The students must choose a strategy and coordinate between various explanations in their tasks. They must decide what the desired level of detail is and how to present the sequence of actions or logical conditions of the system of actions	**20**
Level 4: Reflection on thinking: The students have to test the properties of the final product compared with a set of goals, explain the conclusions to challenges during the development process, and advise on improving the planning and construction process.	**9**

The subject selection was to meet the overall level of creative thinking. Therefore, the subjects selected for interviewing are those that meet the criteria. Based on the number of students at each level in Table 1, one student was selected to interview. The selected subjects were interviewed based on students' oral and written communication skills. Researchers will analyze the answers based on APOS theory with a reflective abstraction construction mechanism from each student's answer at each level.

Data Collection

A holistic rubric was used in the study. A holistic rubric is a rubric that considers criteria simultaneously, requiring one decision on the quality of work across all criteria at once. In addition, the description of the level of performance uses the desired descriptive language (Dawson, 2015; Brookhart, 2018). This rubric was made before the test was given to the subject of the study. This rubric is suitable for emphasizing the quality of students' overall work, so it follows this study.

All subjects were given test questions based on the criteria for the level of creative thinking and then selected for an interview. Researchers saw and observed the activities of students completing the test questions; furthermore, the researchers interviewed the selected research subject. The data in this study was obtained from the written answers obtained by students obtained after they finished the problem of understanding the concept of numbers at the roots of quadratic equations and interviews conducted by researchers. In this study, the results of the students' written tests were collected, and each interview was recorded as a video.

Data Analysis

The overall results of the video recording were carefully viewed and analyzed by attributing the relationship between the results of the student's written test and the corresponding related literature. The steps taken in the data analysis process include (1) transcribing the collected data; (2) reviewing the available data from the test results and interview transcripts; (3) conducting data reduction by selecting, focusing, and classifying similar data, and then simplifying it by removing unnecessary things; (4) presenting data on research results; (5) analyzing students' thought processes in understanding the concept of numbers in solving the roots of quadratic equations using the framework of abstraction-reflective construction mechanisms based on APOS Theory, (6) verifying findings and drawing conclusions.

According to Piaget (in Dubinsky, 2002), reflective abstraction causes the development of cognitive structures. Furthermore, Dubinsky (2002), Meel (2003), Arnawa et al. (2007), Maharaj (2010), Arnon, Cottrill, et al. (2014), and Utami et al. (2019) stated that the reflective abstraction mechanism has six constructions: interiorization, coordination, reversal, encapsulation, de-encapsulation, and generalization. Table 2 shows the definition of construction in this study.

Table 2. Definition of Construction in Research

Construction Process	Construction Definition
Interiorization	Information-gathering thinking (checking discriminant whether the square of a natural number is or not from a known quadratic equation to determine which way is more effective and efficient to use to find solutions).
Coordination	Constructing a new process from two or more other processes (determining the best method/formula from previous knowledge, such as looking for discriminant values, whether $D<0$, $D>0$, or $D=0$, to determine the best formula/way to obtain solutions or conclusions).

Reversal	Reverse construction (substituting the coefficient values of the variables and constants in the quadratic equation into the "abc formula" found to find the roots of the quadratic equation after knowing the discriminant value).
Encapsulation	Constructing mental objects from mental processes (presenting answers to the roots of quadratic equations precisely)
De-encapsulation	Checking the mental objects produced through mental processes (checking the results of answers/solutions/mental objects with previously passed processes, i.e., mental processes)
Generalization	Applying the scheme to a wider collection of phenomena (inferring the resulting solution of the roots of the quadratic equation that cannot be solved using a system of real numbers).

Researchers made observations on pupils as they completed test questions. The following step is an interview with the chosen research topic. Written responses to the challenge of understanding the notion of numbers in calculating the roots of quadratic equations and interviews done by researchers provided the research data. When performing data analysis, triangulation is used to assure validity and dependability. This triangulation was accomplished using data from student-written tests and the display of data from video recordings.

The video interview and student written test results were thoroughly examined to discover the mechanism of formation of reflective abstractions based on APOS Theory in grasping the concept of numbers in solving quadratic equation roots. After each level of creative thinking, the ability has been completed. Data processing begins following that level in carrying out the thinking process based on categories of characteristics, the mechanism of construction of reflective abstraction by APOS theory in understanding the concept of numbers on the completion of the roots of quadratic equations. One of the most crucial steps in this study is categorizing, which aids in appropriate interpretation. This categorization is derived from a literature survey, identification of each test answer, and conversations with authors describing comparable concepts in the literature.

RESULT AND DISCUSSION

Research Results

The explanation that follows each response demonstrates the method of forming reflective abstractions based on the APOS Theory in understanding the concept of numbers at the roots of the equations. The subject denoted by S1 is the first level. Here is Subject 1's (S1) initial answer excerpt in Figure 2, in which S1 shifts the equation to the other side so that nothing remains on the other side, but S1 does not equate to 0 on the equation. S1 made a mistake at this stage where it should be from $x^2 - 4x = -8$ to be $x^2 - 4x + 8 = 0$. When the researchers interviewed S1, S1 stated that owing to the rush of work; he did not write down 0 after the equal sign (equivalence). S1 believed that what was written was correct enough to proceed to the following procedure, namely the search for Discriminant (D), even though $x^2 - 4x + 8$ it was not yet correct where it should be $x^2 - 4x + 8 = 0$, as indicated previously. The final answer will be affected if answer descriptions are written accurately and clearly according to the criteria. S1 has performed an interiorization procedure, attempting to extract information in the question by examining the discriminant, as illustrated in Figure 2. According to what was written on the answer sheet, the researchers matched the answers with the interview findings and found that S1's answer is $\mathrm{D} = -16$.

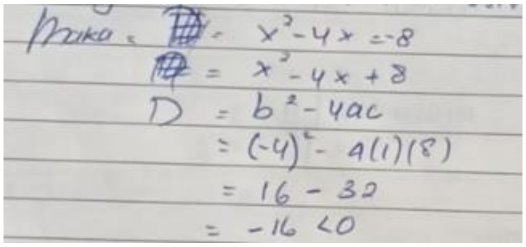

Figure 2 Piece of S1's Answer

Furthermore, S1 tried to continue the coordination process by checking the Discriminant (D) in the interiorization process to determine the most appropriate formula/way of discovering the set of solutions or conclusions of answers, D<0 value (discriminant less than 0). Based on the results of interviews with researchers, S1 concludes in this section that $\mathrm{D} < 0$, but the writing on the answer sheet is still incorrect where it should be after getting $\mathrm{D} = -16$, then it should be followed by writing $\mathrm{D} < 0$. Up to this process, S1 has made two mistakes in writing equivalency in creating an equation, but the researchers believed that S1 understands this concept based on interviews with S1.

S1, here, accomplished encapsulation, that is, construction via discriminant checking to the conclusion. S1 did not reverse the encapsulation process in the form of discriminant values to the generalization stage by substituting the coefficient values of the variables and constants in the quadratic equation into the "abc formula." At this step of generalization, S1 specified the conclusion that there is no real root, which means that a system has no solution if the D is negative (not real). The researchers asked S1 whether S1 believed that the result of D, a negative integer, was not a real number (because the answer is written "not real"). S1 replied doubtfully, with a yes answer. The researchers then reinforced the answer, and S1 confirmed it, i.e., if $\mathrm{D} < 0$, then there is no solution in the real number system, as shown in Figure 3.

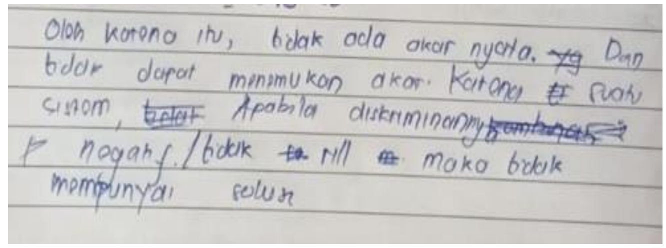

Figure 3 Pieces of S1's Answer

The second level is the subject symbolized by S2 which means that students are at the level of creative thinking level 2. The following preliminary answer to Subject 2 (S2) is shown in Figure 4 below. S2 moved the equation to the other side so that there is nothing left on the other side; there are no remaining variables on the other side, and it is written 0 next to the equal sign, i.e., from $\mathrm{x}^2 - 4\mathrm{x} = -8$ to be $\mathrm{x}^2 - 4\mathrm{x} + 8 = 0$. S2 has carried out an interiorization process, which entails digging up the information in the problem by checking the discriminant, as shown in Figure 4. The researchers checked the answers with the results of the interview that S2 got the results $\mathrm{D} = -16$, according to what was written on the answer sheet.

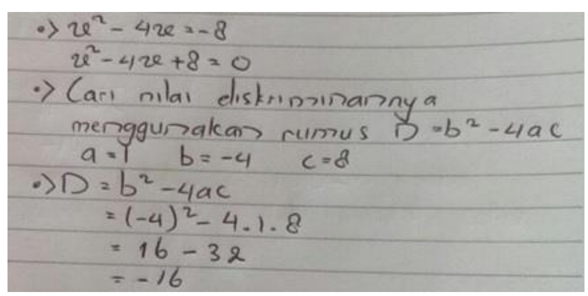

Figure 4 Piece of S2's Answer

Furthermore, S2 continued the coordination process by examining the Discriminant (D), as shown in Figure 4, where the value of D<0 (discriminant less than 0) is written in the final answer to the conclusion in Figure 6. First, the researchers asked S2 why there is no description of the value of D<0 (discriminant less than 0) in this section. Then S2 replied that to shorten the answer, it is only written at the end, such as in Figure 6.

S2 also performed encapsulation, that is, construction through discriminant checking to the conclusion, but did not find the equation of the square roots, as illustrated in Figure 4. Next, S2 performed de-encapsulation by explaining that if the quadratic function $y = ax^2 + bx + c$ is depicted in Cartesian coordinates, if $b^2 - 4ac < 0$, then the quadratic equation has no real roots since the graph does not contact or intersect the axis, as shown in Figure 5. S2 did not check for the results of the problem's roots in the quadratic equation. Hence it does not describe whether the solution set is a complex number. S2 merely verified the discriminant. S2 has carried out the de-encapsulation process by comparing the results of the answer with the process it goes through, namely the encapsulation process. S2 did not reverse the procedure of solving the quadratic equation by replacing the coefficient values of the variables and constants with the "abc formula."

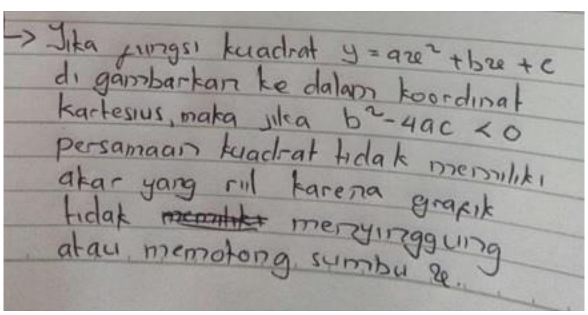

Figure 5 Piece of S2's Answer

When the researchers asked S2 what the intended solution of the one written in the answer was, S2 replied that it was the solution of the real number system. S2 stated that he was in a rush when he wrote it down. Hence it was only partially completed to the point of "having no solution." Furthermore, as a result of the de-encapsulation process, S2 performed a generalization process, determining that the solution arising from the roots of the quadratic equation could not be solved using the real number system, as shown in Figure 6.

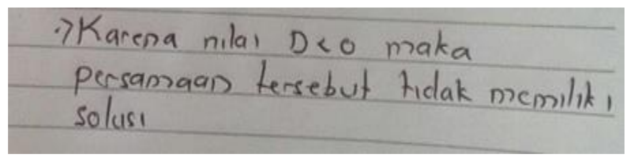

Figure Piece of S2's Answer

The third level is the subject symbolized by S3 which means that students are at the level of creative thinking level 3. S3 moved the equation to the other side so that there is nothing left on the other side; there is no remaining variables on the other side, and it is written 0 next to the equal sign i.e., from $x^2 - 4x = -8$ to be $x^2 - 4x + 8 = 0$, as in Figure 7.

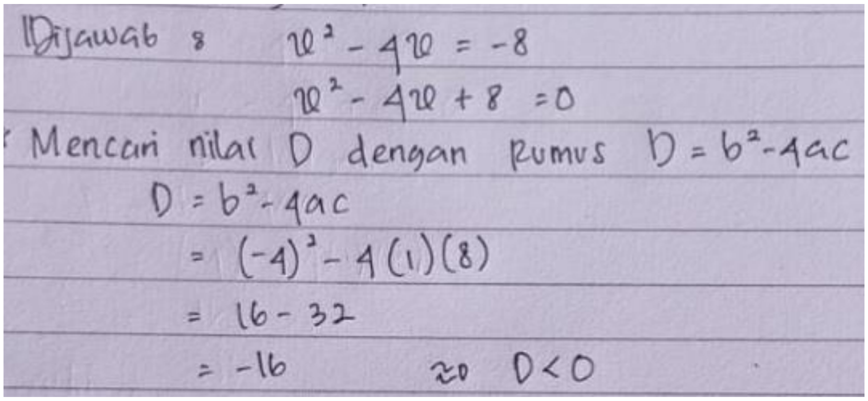

Figure 7 Piece of S3's Answer

S3 has also undergone interiorization, which entails digging up information in the matter by examining the discriminant (D). Based on the interview with S3, which is due to D<0, it can immediately employ methods other than factoring to obtain the results of quadratic equation roots. Furthermore, S3 continued the coordination process by selecting the most appropriate way/formula of previous knowledge to obtain the outcome of the quadratic equation's roots from the previously checked determinant, the abc formula, as shown in figure 8 below. Based on the interview results, S3 performed the encapsulation process by displaying the value of D<0 (discriminant less than 0), indicating that the equation has no solution in the real number system, so S3 continued to the reversal process, which is to substitute the value of the coefficients of the variables and constants in the quadratic equation into the "abc formula" to find solutions from quadratic equations (roots of quadratic equations). The value of the roots of the equation is $x_{12} = \frac{4 \pm \sqrt{-16}}{2}$. Here, S3 has performed de-encapsulation by comparing the results of the answers to the process by which it passes through encapsulation and reversal to generate the correct answer, leading to the conclusion.

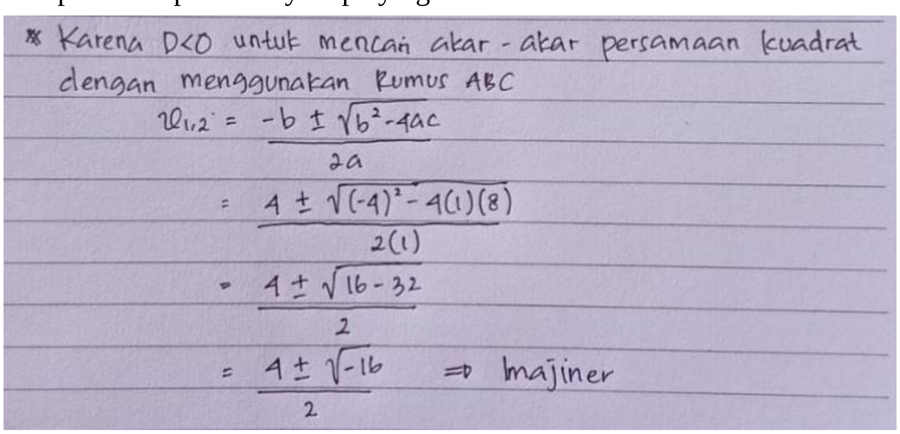

Figure 8 Piece of S3's Answer

Furthermore, as a result of the de-encapsulation process, S3 performed a generalization process, inferring that the resulting solution of the roots of the quadratic equation cannot be solved using the real number system solution, as shown in Figure 9. When the researchers asked about the 'imaginary' written by S3 in Figure 8, S3 responded that the solution of the quadratic equation's roots was imaginary. Based on the written responses and interview results, S3 concluded that the equation result did not have a real number solution in the presence of a cross-check between the D<0 search and the quadratic equation roots.

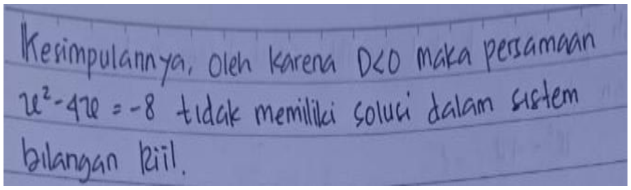

Figure 9 Piece of S3's Answer

The fourth level is the subject symbolized by S4 which means that students at the level of creative thinking level 4, where in this study, the subject is symbolized by S4. S4 moved the equation to the other side so that there is nothing left on the other side; there is no remaining variables on the other side, and it is written 0 next to the equal sign, i.e., from $x^2 - 4x = -8$ to be $x^2 - 4x + 8 = 0$. S4 has carried out an interiorization process, which entails digging up the information in the problem by checking the discriminant (D). Based on interviews conducted with S4, because D < 0, it is possible to directly use other methods besides factoring to find the results of the roots of the quadratic equation.

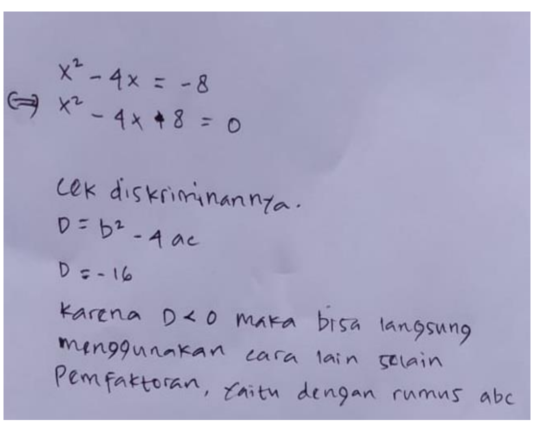

Figure 10 Piece of S4's Answer

Furthermore, S4 continued the coordination process by determining the most appropriate way/formula of previous knowledge to find the result of the roots of the quadratic equation from the previously checked determinant, namely the abc formula. Finally, based on the results of the interview conducted, S4 carried out the encapsulation process by showing the value of D<0 (discriminant less than 0) and immediately checked using the "abc formula", as written in Figure 10.

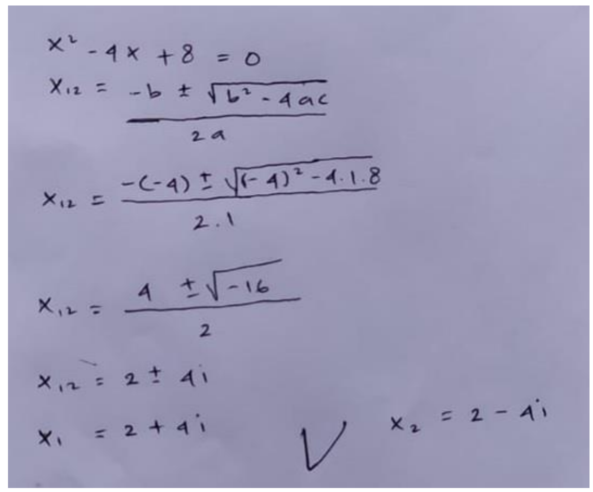

Figure 11 Piece of S4's Answer

Figure 11 shows that S4 carried out the reversal process to substitute the coefficient values of the variables and constants in the quadratic equation into the "abc formula" to find solutions to the quadratic equations (the roots of the quadratic equation). The value of the roots of the equation is $x_{12} = \frac{4 \pm \sqrt{-16}}{2}$. Here, the subject has carried out a de-encapsulation process by checking the results of the answers with the process through which it passes through encapsulation and reversal to produce the right answer leading to the conclusion.

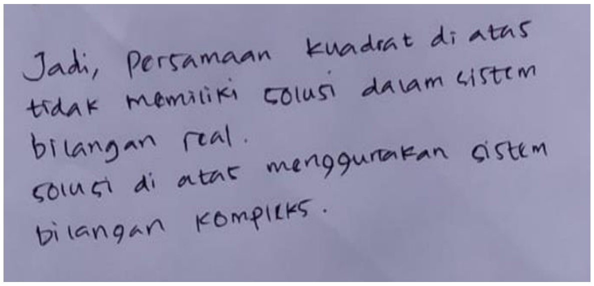

Figure 12 Piece of S4's Answer

Furthermore, from the encapsulation and reversal process, S4 carried out a generalization process, which is to infer that the solution resulting from the roots of the quadratic equation cannot be solved using the real number system solution by providing a more detailed description of the specialization process, that is, the solution can be solved using a complex number system. This is shown as shown in Figure 12. The specialization process was carried out by pursing the results of $x_{12} = \frac{4\pm\sqrt{-16}}{2}$This is by revealing the results of the researchers' interview with S4 regarding the reason. The value of the roots of the equation is x_1 and x_2 shown in the form of complex numbers $x_{12} = a \pm bi$ is $x_{12} = 2 \pm 4i$. In response to a question posed by the researchers during an interview with S4, who had not yet taken complex analysis courses at the introductory level of higher education, S4 stated that his understanding of complex numbers had been independently acquired from numerous sources.

Discussion

According to the findings of Dubinsky (2002), Meel (2003), Arnawa et al. (2007), Maharaj (2010), Arnon, Cottrill, et al. (2014), and Cetin & Dubinsky (2017), the mechanism of reflective abstraction has six constructions: interiorization, coordination, reversal, encapsulation, de-encapsulation, and generalization. Meanwhile, Utami (2019) identifies five phases of reflective abstraction construction in students in the lowest mental model, namely pre-initials, including interiorization, coordination, reversal, encapsulation, and generalization. In this study, the researchers developed seven mechanisms of reflective abstraction from prior research conducted by De Bono, adding a specialization construction process following the generalization construction process in students who are at the 4th level of creative thinking or the highest level of creative thinking.

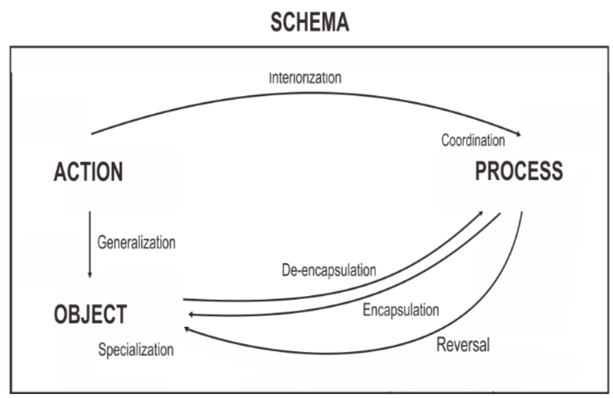

Figure 13 Construction Mechanisms of Reflective Abstraction based on APOS Theory in Understanding the Concept of Numbers at the Roots of Quadratic Equations

The following is a description of the building of reflective abstractions based on APOS Theory to comprehend the concept of numbers and solve mathematical issues involving the roots of quadratic equations in Table 3.

Table 3. Definition of Construction Based on the Research Conducted by Suriyah., et. al.

Construction Process	Construction Definition
Interiorization	Information-gathering thinking (checking discriminant whether the square of a natural number is or not from a known quadratic equation to determine which way is more effective and efficient to use to find solutions).

Coordination	Constructing a new process from two or more other processes (determining the best method/formula from previous knowledge, such as looking for discriminant values, whether D<0, D>0, or D=0, to determine the best formula/way to obtain solutions or conclusions).
Reversal	Reverse construction (substituting the coefficient values of the variables and constants in the quadratic equation into the "abc formula" found to find the roots of the quadratic equation after knowing the discriminant value).
Encapsulation	Constructing mental objects from mental processes (presenting answers to the roots of quadratic equations precisely)
De-encapsulation	Checking the mental objects produced through mental processes (checking the results of answers/solutions/mental objects with previously passed processes, i.e., mental processes)
Generalization	Applying the scheme into a wider collection of phenomena (inferring the resulting solution of the roots of the quadratic equation that cannot be solved using a system of real numbers).
Specialization	Capable of providing precise descriptions/descriptions in absorbing schemes for a larger set of phenomena (inferring solutions from the roots of quadratic equations that can be solved using complex number systems)

Based on Figure 13 and the explanation in Table 3, the construction of reflective abstraction based on APOS Theory for comprehending the concept of numbers in solving mathematical problems involving the roots of quadratic equations at each level of students' creative thinking is elaborated. As depicted in Figure 14, the reflective abstraction construction mechanism for students with the 1st level of creative thinking is described. APOS Theory has abstracted from action to process, which is consistent with prior research by Scheiner (2016), who focuses on "abstraction from action" and rejects the concept of "abstraction from objects". According to the results of this study, however, students at the 1st level of creative thinking do not de-encapsulate the mental objects produced by mental processes (that is, they do not compare the results of answers/solutions/mental objects with the previous process, namely mental processes). In addition, students do not engage in reverse construction processes, such as substituting the coefficient values of the variables and constants in the quadratic equation into the "abc formula" discovered to find the roots of the quadratic equation after knowing the discriminant value. This is consistent with the application of APOS theory, which is utilized to determine how mathematicians perceive the mathematical concepts they teach and to determine the strength and stability of the mathematical constructions (Arnon, 2014; Arnon & Cotril, 2014). In this study, the mechanism of the reflective abstraction construction process based on the APOS theory is depicted in Figure 14 for students with the 1st level of creative thinking, where there are weaknesses because students do not perform the de-encapsulation and reversal processes. One can determine the relationship between mathematics and students based on these weaknesses (Chaabi et al., 2019). The hope is that it will be able to make evaluations for future learning by providing students with more appropriate models, strategies, learning media aids, and others (Yusnaeni et al., 2017).

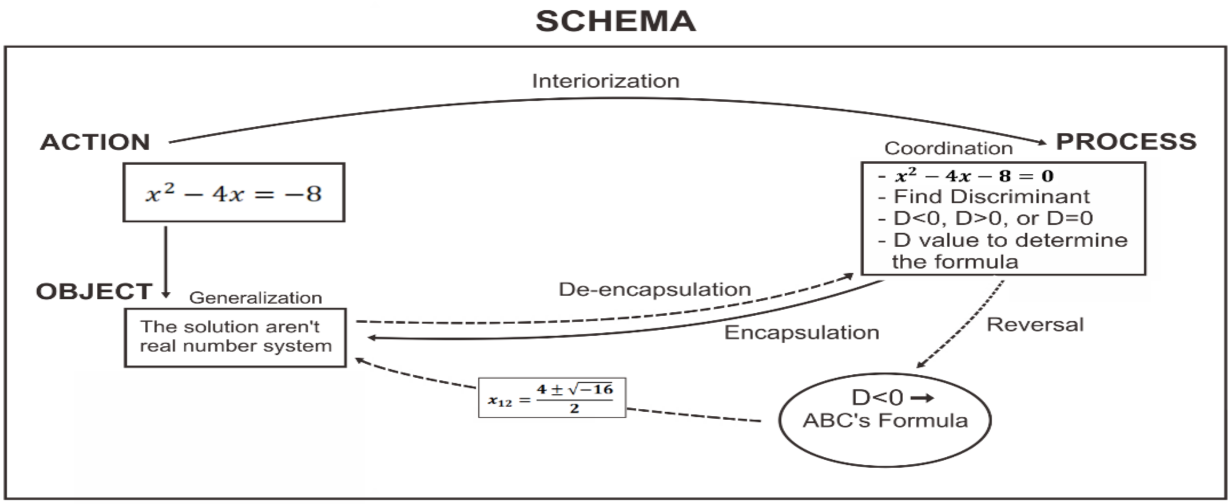

Figure 14 Scheme of Reflective Abstraction Mechanisms in Students with the 1st Level of Creative Thinking

Figure 15 depicts the process of the reflective abstraction construction mechanism for students with a level 2 creative thinking ability. Students have completed the interiorization, coordination, encapsulation, and de-encapsulation processes. Students do not reverse the process by substituting the coefficient values of the variables or constants in the quadratic equation into the "abc formula" to find the roots of the quadratic equation after knowing the discriminant value. The generalization process involves examining the discriminant so that the subject can determine that the solution is not a real number. In this case, students with a level 2 creative thinking level do not perform a reversal process. Teachers can carry out a better learning process by understanding the condition of their students, as is the case here (Utami, Usodo, & Pramudya, 2019).

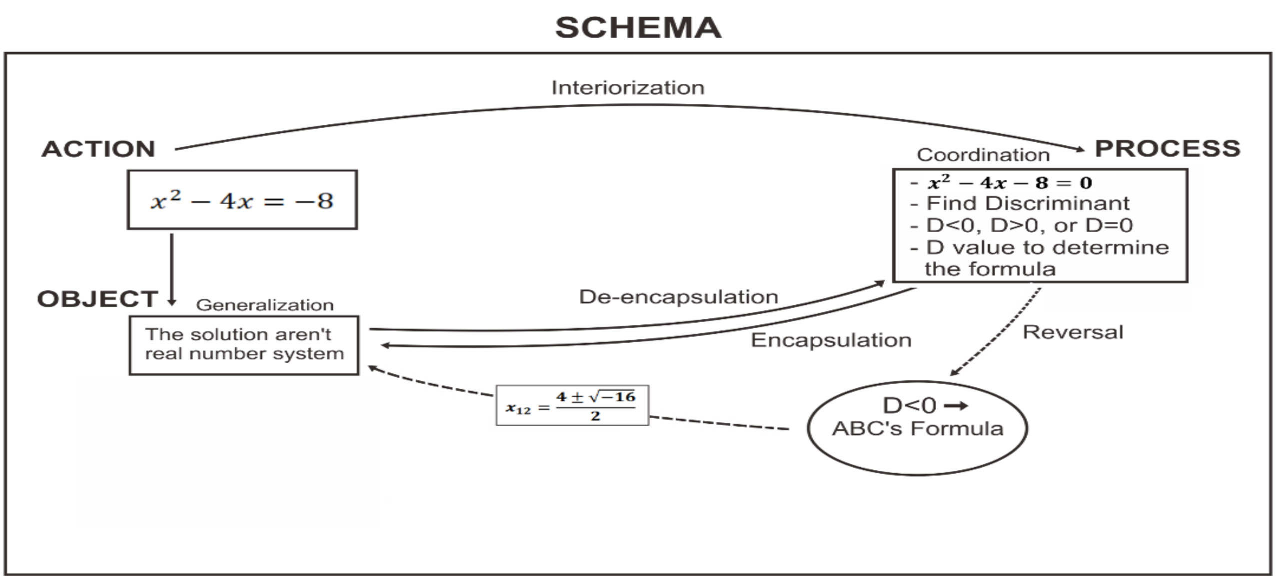

Figure 15 Scheme of Reflective Abstraction Mechanisms in Students with the 2nd Level of Creative Thinking

Figure 16 depicts the process of reflective abstraction construction mechanisms in students with a level 3 of creative thinking. Students engage in constructive mechanisms of reflective abstractions ranging from interiorization to generalization (Dubinsky, 2002; Meel, 2003; Arnawa et al., 2007; Maharaj, 2010; Arnon, Cottrill, et al., 2014; Cetin & Dubinsky, 2017).

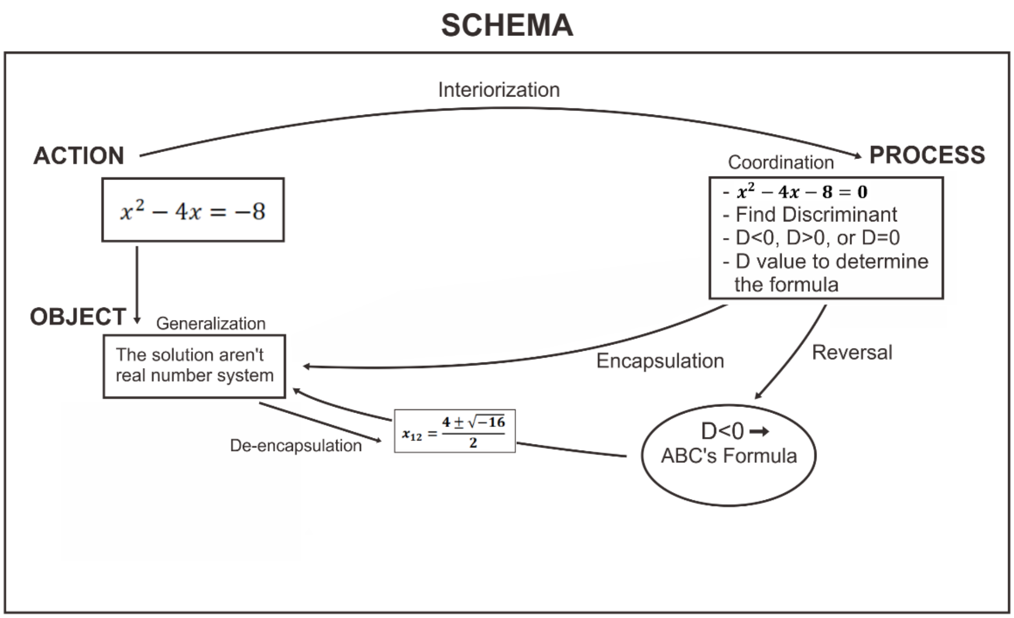

Figure 16. Scheme of Reflective Abstraction Mechanisms in Students with the 3rd Level of Creative Thinking

Figure 17 depicts the process of reflective abstraction construction mechanisms in students with a level 4 of creative thinking. Students engage in more constructive mechanisms of reflective abstraction than in previous studies (Dubinsky, 2002; Meel, 2003; Arnawa et al., 2007; Maharaj, 2010; Arnon, Cottrill, et al., 2014; Cetin & Dubinsky, 2017), indicating that there is a process of specialization following the process of generalization. At this level, there are also research findings in the specialization process. For example, students can infer solutions from the roots of quadratic equations that can be solved using a complex number system, so in this case, students can provide descriptions precisely in applying the scheme to a broader collection of phenomena known as specialization in this study. Based on the previous data display, students can independently dig from various literature to carry out construction in solving mathematical problems with the correct concepts before receiving detailed material from lecturers/educators. Creativity in education is required to open up new avenues that enhance learning quality (Beetlestone, 2013). Similarly, Yusnaeni et al. (2017) claim that giving students the responsibility to learn independently in research such as those conducted by students at level 4 will help them improve their creative thinking skills.

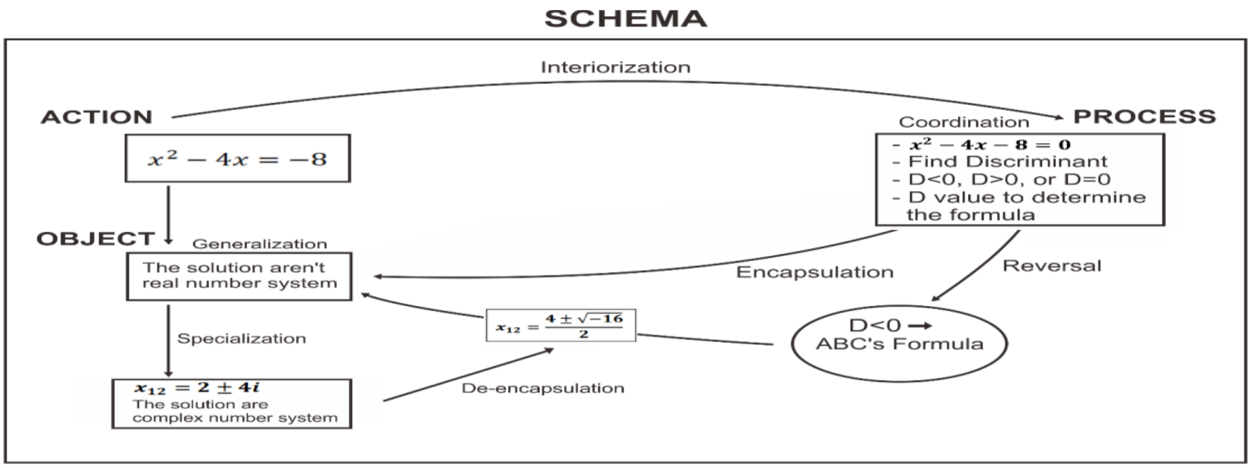

Figure 17 Scheme of Reflective Abstraction Mechanisms in Students with the 4th Level of Creative Thinking

CONCLUSION

According to the findings of this study, there are seven mechanisms of the process of building reflective abstractions in understanding the concept of complex functions. Interiorization, coordination, reversal, encapsulation, de-encapsulation, generalization, and specialization are the seven mechanisms of the reflective abstraction construction process. When students check the discriminant, whether the square of the natural number or not of the known quadratic equation, they perform the interiorization process to determine which way is more effective and efficient to use to find a solution. When students determine the most appropriate method/formula from previous knowledge, namely looking for discriminate values whether D<0, D>0, or D = 0 to make it easier to determine the most appropriate formula/way to find the set of solutions or conclusions of answers, they perform coordination process. When students can substitute the coefficient values of variables and constants in the quadratic equation into the "abc formula" found to find the roots of the quadratic equation after the discriminant value is known, they perform the reversal process. When students precisely present the answers to quadratic equation roots, they perform the encapsulation process. When students compare the results of answers/solutions/mental objects to the previous process, namely the mental process, they perform the de-encapsulation process. When students infer the resulting solution of the roots of the quadratic equation that cannot be solved using the real number system, they perform the generalization process.

In addition, this study also finds a mechanism for the seventh reflective abstraction construction process, namely "specialization" following "generalization." After students provide solutions to a quadratic equation with discriminant checking (D<0), the specialization process can be observed because the solution is not a real number system. Students then continue substituting the quadratic equation into the ‘abc formula’ to determine the quadratic equation's roots. The final result demonstrates that the quadratic equation D<0 has no solution in the real number system, and the "specialization" process is completed by stating that the solution lies in the complex number system, where the real part is R(z), and the imaginary part is I(z) for students with the highest level of creative thinking ability.

For this reason, the researchers recommend additional research to explore in greater depth the mechanism of the reflective abstraction construction process for students with other abilities; Is it also possible to develop the mechanism of the reflective abstraction construction process for students with other abilities? Considering that, following the mechanism of reflective abstraction construction, generalization has a process of specialization for students with the most advanced creative thinking skills. Then, are students with the highest level of creative thinking ability and appropriate and specific solutions with a high level of learning autonomy? Given that it turns out that students at the highest level of creative thinking ability can complete solutions correctly and specifically (more than those that teachers have not taught).

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