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Modelling to Carbon Fiber Data: Using Anew Two parameters Inverse Rayleigh Model

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Abstract

In this paper, we propose the type II truncated Frèchet inverse Rayleigh (TIITFIR) distribution, which has two parameters that are extended from the inverse Rayleigh (IR) distribution. The new model may be more adaptable. Some of its basic qualities are presented. Maximum likelihood (ML) estimation was used to examine TIITFIR parameter estimators. The TIITFIR parameters' asymptotic confidence intervals are investigated. We evaluate the simulation in order to investigate the theoretical outcomes. The new model's significance and adaptability are evaluated using a single real-world data set.

Keywords: Type II truncated inverse Frèchet-G family, inverse Rayleigh distribution, moments, maximum likelihood, simulation.

1. Introduction

The inverse Rayleigh (IR) model is proposed by Trayer (1964) it is used in model reliability and survival data sets. After that, IR model was championed by Voda (1972). He studied its properties and maximum likelihood estimator of the unknown parameter.

Lot of works have been studied in the literature on IR distribution. Hassan et al. (2010) estimated the parameters using classical and Bayesian estimation methods. Beta IR model was introduced by Leao et al. (2013), Ahmed et al. (2014) introduced a generalization of the IR model, modified IR distribution proposed by Khan (2014), Rehman and Dar (2015) proposed exponentiated IR model, Khan and King (2015) studied transmuted modified IR distribution, Haq (2016a) introduced transmuted exponentiated IR distribution, Kumaraswamy exponentiated IR distribution was studied by Haq (2016b), Elgarhy and Alrajhi (2019) introduced odd Frèchet IR distribution, Mohammed and Yahia (2019) studied Type II Topp Leone IR and Yahia and Mohammed (2019) studied Type II Topp Leone generalized IR.

The probability density function (pdf) and *cumulative distribution function (cdf)* of IR distribution are given

$$g(x:\alpha) = \frac{2\alpha}{x^3} e^{-\frac{\alpha}{x^2}} \quad , \quad x,\alpha > 0, \qquad (1)$$

and

$$G(x:\alpha) = e^{-\frac{\alpha}{x^2}} \quad , \quad x,\alpha > 0, \qquad (2)$$

Several academics interested in produced families of distributions have recently published their findings. The truncated models were utilized as generators by some of the researchers. Abid and Abdulrazak (2017) studied truncated Fréchet-G, Najarzadegan et al. (2017) proposed the truncated Weibull-G, Bantan et al. (2019) studied the truncated inverted Kumaraswamy -G, Aldahlan (2019) introduced the type II truncated Fréchet –G (TIITF-G) and it has the following pdf and cdf

$$f(x;\theta,\xi) = \theta e g(x;\xi)(1 - G(x;\xi))^{-\theta-1} e^{-(1-G(x;\xi))^{-\theta}} \quad , \quad x \in R, \quad \theta > 0. \qquad (3)$$

and

$$F(x;\theta,\xi) = 1 - ee^{-\left(1-G(x;\xi)\right)^{-\theta}}, x \in R, \quad \theta > 0. \qquad (4)$$

The survival function (sf), hazard rate function (hrf) and quantile function *Q(u)* of the TIITF-G family is

$$R(x;\theta,\xi) = ee^{-\left(1-G(x;\xi)\right)^{-\theta}},$$

$$h(x:\theta,\xi) = \theta g(x;\xi)(1-G(x;\xi))^{-\theta-1},$$

and

$$Q_{(u)} = 1 - \left[ln\left(\frac{e}{1-u}\right)\right]^{\frac{-1}{b}}.$$

The primary purpose of this study is to define a new lifespan model known as the TIITFIR model. We expect that it will be exceedingly adaptable and attract a broader range of applications in various fields of study. The following is how this paper is organized. Sections 2 and 3 define and investigate the TIITFIR, as well as compute its structural characteristics. In Section 4, the ML approach is used to drive the model parameter estimators. In Section 5, simulations are run to get estimates of the model parameters of the TIITFIR model. Section 6 examines one real-world data set. Finally, Section 7 contains the findings.

2. The TIITFIR Model

The pdf and cdf of TIITFIR distribution are investigated by entering (1) and (2) into (3) and (4), as below

$$f(x;\theta,\alpha) = \frac{2\alpha\theta e}{x^3} e^{-\frac{\alpha}{x^2}}(1-e^{-\frac{\alpha}{x^2}})^{-\theta-1} e^{-(1-e^{-\frac{\alpha}{x^2}})^{-\theta}} \qquad x,\alpha,\theta > 0, \qquad (5)$$

and

$$F(x;\varphi) = 1 - ee^{-(1-e^{-\frac{\alpha}{x^2}})^{-\theta}} \qquad x,\alpha,\theta > 0. \qquad (6)$$

Also, sf, hrf, reversed hrf and cumulative hrf of X are provided with

$$R(x;\varphi) = ee^{-(1-e^{-\frac{\alpha}{x^2}})^{-\theta}},$$

$$h(x;\varphi) = \frac{2\alpha\theta}{x^3} e^{-\frac{\alpha}{x^2}}(1-e^{-\frac{\alpha}{x^2}})^{-\theta-1},$$

$$\tau(x;\varphi) = \frac{\frac{2\alpha\theta e}{x^3} e^{-\frac{\alpha}{x^2}}(1-e^{-\frac{\alpha}{x^2}})^{-\theta-1} e^{-(1-e^{-\frac{\alpha}{x^2}})^{-\theta}}}{1-ee^{-(1-e^{-\frac{\alpha}{x^2}})^{-\theta}}},$$

and

$$H(x;\varphi) = (1-e^{-\frac{\alpha}{x^2}})^{-\theta} - 1.$$

Figures 1, 2, 3 and 4 shows some cdf, pdf, sf and hrf plots of *TIITFIR* for various values of parameters φ.

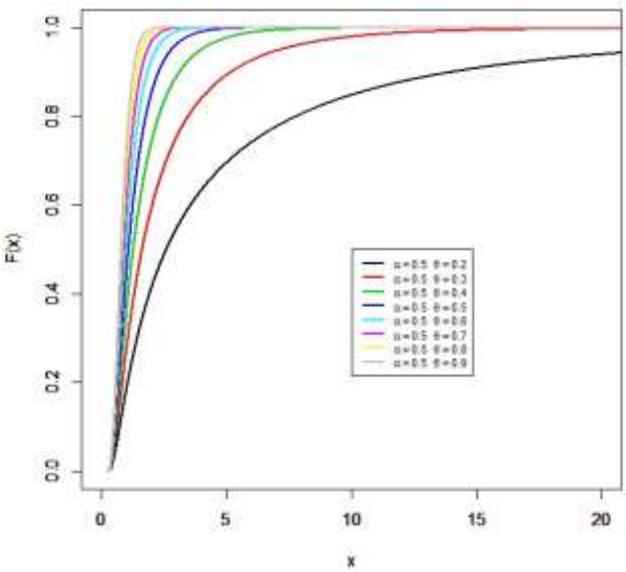

Figure 1: Plots of the cdf of the TIITFIR distribution.

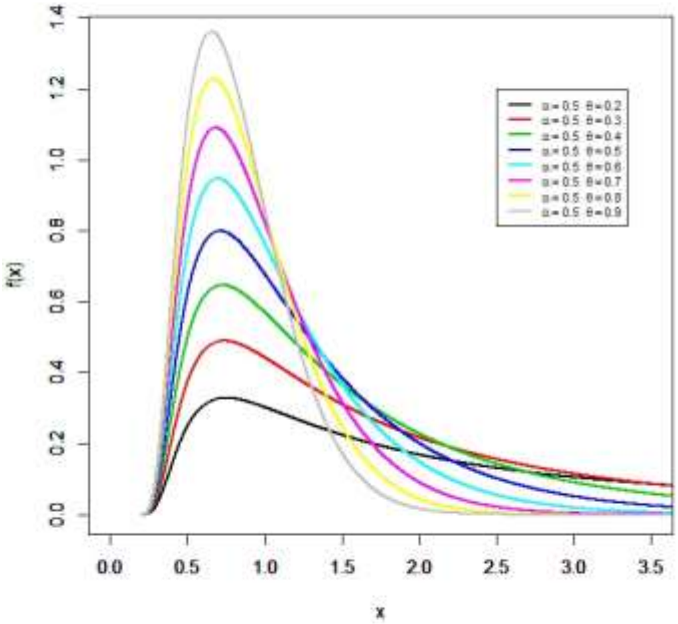

Figure 2: Plots of the pdf of the TIITFIR distribution.

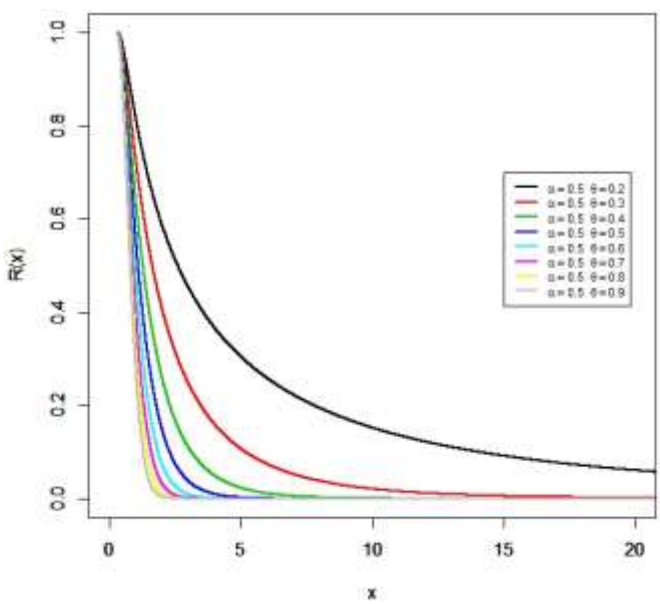

Figure 3: Plots of the sf of the TIITFIR distribution.

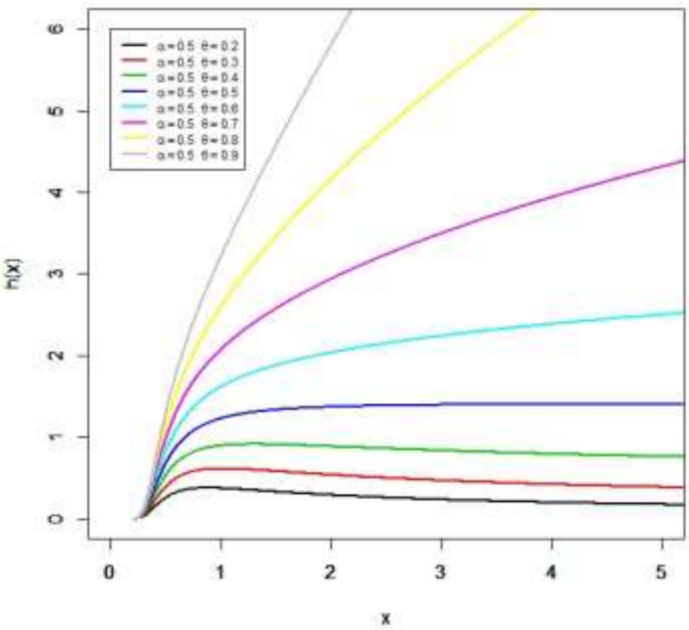

Figure 4: Plots of the hrf of the TIITFIR distribution.

Figures 2 and 4 show that the TIITFIR model's pdf may be uni-model and right skewed. In addition, the hrf of the TIITFIR model may increase.

3. Statistical Features

Some features of the TIITFIR distribution are computed in this section.

3.1 Quantile and Median

The quantile function, say $Q(u) = F^{-1}(u)$ of X is provided with

$$Q(u) = \sqrt{\frac{-\alpha}{ln\left\{1-\left[ln\left(\frac{e}{1-u}\right)\right]^{\frac{-1}{\theta}}\right\}}}, \qquad 0 < u < 1. \qquad (7)$$

In particular, by allowing u = 0.5, the median may be calculated from (7). In other words, the median (M) equals

$$M = \sqrt{\frac{-\alpha}{ln\left\{1-\left[ln(2e)\right]^{\frac{-1}{\theta}}\right\}}}.$$

3.2 Useful Expansion

The pdf representations for the TIITFIR distribution are generated in this subsection. Aldahlan (2019) expressed (3) as

$$f(x) = \sum_{k=0}^{\infty} \eta_k g(x,\xi) G(x,\xi)^{k+1}, \qquad (8)$$

where

$$\eta_k = \sum_{j=0}^{\infty} \frac{be(-1)^{k+j}}{j!} \binom{b(j+1)+k}{k}.$$

We may rewrite the pdf (5) of TIITFIR as a linear combination of IR distributions by entering (1) and (2) into (8).

$$f(x) = \sum_{k=0}^{\infty} \frac{w_k}{x^3} e^{-\frac{\alpha(k+2)}{x^2}}, \qquad (9)$$

where $w_k = 2\alpha\eta_k$.

3.3 Moments

The rth moment of the pdf (5) is provided with

$$\mu_r' = E(X^r) = \int_{-\infty}^{\infty} x^r f(x;\varphi)dx. \quad (10)$$

Substituting (9) into (10) we get

$$\mu_r' = E(X^r) = \sum_{k=0}^{\infty} w_k \int_0^{\infty} x^{r-3} e^{-\alpha(k+2)x^{-2}} dx.$$

Let $y = x^{-2}$, then,

$$\mu_r' = \sum_{k=0}^{\infty} \frac{w_k}{2} \int_0^{\infty} y^{\frac{-r}{2}} e^{-\alpha(k+2)y} dy,$$

then, μ_r' becomes

$$\mu_r' = \sum_{k=0}^{\infty} \frac{w_k \Gamma(1-\frac{r}{2})}{2[\alpha(k+2)]^{1-\frac{r}{2}}} , \quad r<2.$$

The moment generating function of TIITFIR model is

$$M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(X^r) = \sum_{r,k=0}^{\infty} \frac{t^r}{r!} \frac{w_k \Gamma(1-\frac{r}{2})}{2[\alpha(k+2)]^{1-\frac{r}{2}}} , \quad r<2.$$

The incomplete moments, say $\varpi_s(t)$, is given by

$$\varpi_s(t) = \int_0^t x^s f(x;\varphi)dx = \sum_{k=0}^{\infty} w_k \int_0^t x^{s-3} e^{-\alpha(k+2)x^{-2}} dx .$$

Then,

$$\varpi_s(t) = \sum_{k=0}^{\infty} w_k \frac{\nu\left(1-\frac{s}{2}, \alpha(k+2)t^{-2}\right)}{2\left(\alpha(k+2)\right)^{1-\frac{s}{2}}} , \quad s<2.$$

Where $\nu(s,t) = \int_0^t x^{s-1} e^{-x} dx$ is the lower incomplete gamma function.

Further, the conditional moments, say $\Delta_s(t)$, is given by

$$\Delta_s(t) = \int_t^{\infty} x^s f(x;\varphi)dx = \sum_{k=0}^{\infty} w_k \int_t^{\infty} x^{s-3} e^{-\alpha(k+2)x^{-2}} dx .$$

Then,

$$\Delta_s(t) = \sum_{k=0}^{\infty} w_k \frac{\Gamma\left(1-\frac{s}{2}, \alpha(k+2)t^{-2}\right)}{2\left(\alpha(k+2)\right)^{1-\frac{s}{2}}} , \quad s<2,$$

where $\Gamma(s,t) = \int_t^{\infty} x^{s-1} e^{-x} dx$ is the upper incomplete gamma function.

3.4 Inequality Measures

The TIITFIR distribution's Lorenz, Bonferroni, and Zenga curves are provided.

$$L_F(x) = \frac{\int_0^t xf(x)dx}{E(X)} = \frac{\sum_{k=0}^{\infty} w_k \frac{\nu\left(\frac{1}{2}, \alpha(k+2)t^{-2}\right)}{2\left(\alpha(k+2)\right)^{\frac{1}{2}}}}{\sum_{k=0}^{\infty} \frac{w_k \sqrt{\pi}}{2[\alpha(k+2)]^{\frac{1}{2}}}},$$

$$B_F(x)=\frac{\int_0^t xf(x)dx}{E(X)F(x)}=\frac{L_F(x)}{F(x)}=\frac{\sum_{k=0}^{\infty} w_k \frac{\nu\left(\frac{1}{2},\alpha(k+2)t^{-2}\right)}{2\left(\alpha(k+2)\right)^{\frac{1}{2}}}}{\left(\sum_{k=0}^{\infty}\frac{w_k\sqrt{\pi}}{2\left[\alpha(k+2)\right]^{\frac{1}{2}}}\right)\left(1-ee^{-\left(1-e^{\frac{-\alpha}{x^2}}\right)^{-\theta}}\right)},$$

and

$$A_F(x)=1-\frac{\mu^-(x)}{\mu^+(x)},$$

where

$$\mu^-(x)=\frac{\int_0^t xf(x)dx}{E(X)}=\frac{\sum_{k=0}^{\infty} w_k \frac{\nu\left(\frac{1}{2},\alpha(k+2)t^{-2}\right)}{2\left(\alpha(k+2)\right)^{\frac{1}{2}}}}{\sum_{k=0}^{\infty}\frac{w_k\sqrt{\pi}}{2\left[\alpha(k+2)\right]^{\frac{1}{2}}}}.$$

And

$$\mu^+(x)=\frac{\int_t^{\infty} xf(x)dx}{1-F(x)}=\frac{\sum_{k=0}^{\infty} w_k \frac{\Gamma\left(\frac{1}{2},\alpha(k+2)t^{-2}\right)}{2\left(\alpha(k+2)\right)^{\frac{1}{2}}}}{ee^{-\left(1-e^{\frac{-\alpha}{x^2}}\right)^{-\theta}}}.$$

4. ML Estimation

Let $X_{(1)},\dots,X_{(n)}$ be observed values from the TIITFIR model with set of parameters $\varphi=(\alpha,\theta)^T$. The total log-likelihood function under complete sample can be expressed as

$$lnL(\varphi)=n\ln(\alpha\theta e)-3\sum_{i=1}^{n}lnx_i-\alpha\sum_{i=1}^{n}\frac{1}{(x_i)^2}-\sum_{i=1}^{n}\left(1-e^{\frac{-\alpha}{(x_i)^2}}\right)^{-\theta}-(\theta+1)\sum_{i=1}^{n}ln\left(1-e^{\frac{-\alpha}{(x_i)^2}}\right).$$

The elements of the score function $U_\varphi=(U_\alpha,U_\theta)$ are given by

$$U_\alpha=\frac{n}{\alpha}-\sum_{i=1}^{n}\frac{1}{(x_i)^2}+\theta\sum_{i=1}^{n}\frac{1}{(x_i)^2}\left(1-e^{\frac{-\alpha}{(x_i)^2}}\right)^{-\theta-1}e^{\frac{-\alpha}{(x_i)^2}}-(\theta+1)\sum_{i=1}^{n}\frac{\frac{1}{(x_i)^2}e^{\frac{-\alpha}{(x_i)^2}}}{1-e^{\frac{-\alpha}{(x_i)^2}}},$$

and

$$U_\theta=\frac{n}{\theta}+\sum_{i=1}^{r}\left(1-e^{\frac{-\alpha}{(x_i)^2}}\right)^{-\theta}ln\left(1-e^{\frac{-\alpha}{(x_i)^2}}\right)-\sum_{i=1}^{n}ln\left(1-e^{\frac{-\alpha}{(x_i)^2}}\right).$$

Then perhaps the ML estimators of the parameters α and b are researched by placing $U_\varphi=0$ and solving them.

5. Numerical Outcomes

This Section includes a simulation study to evaluate the behavior of the estimators in the event of a full sample. Mathematica 9 is used to explore mean square errors (B1), lower limit (B2) and upper bound (B3) of confidence intervals (CI), and average length (B4) of 90% and 95%. Tables 1–8 provide the numerical results based on the entire sample.

Table 1: MLE, B1, B2, B3 and B4 of TIITFIR model for (θ=0.5, α =0.5)

n	MLE	B1	90%			95%		
			B2	B3	B4	B2	B3	B4
30	0.5307	0.0103	0.3825	0.6788	0.2963	0.3542	0.7072	0.3530
	0.5553	0.0248	0.2992	0.8114	0.5121	0.2502	0.8604	0.6102
50	0.5085	0.0045	0.4005	0.6165	0.2160	0.3798	0.6372	0.2574
	0.4939	0.0143	0.3161	0.6717	0.3555	0.2821	0.7057	0.4236
100	0.5130	0.0022	0.4360	0.5901	0.1541	0.4212	0.6048	0.1836
	0.5291	0.0062	0.3944	0.6637	0.2693	0.3686	0.6895	0.3209
200	0.5030	0.0011	0.4500	0.5560	0.1060	0.4399	0.5662	0.1263
	0.4944	0.0029	0.4049	0.5840	0.1791	0.3877	0.6011	0.2134
500	0.5070	0.0005	0.4732	0.5408	0.0676	0.4667	0.5473	0.0806
	0.5070	0.0012	0.4491	0.5649	0.1158	0.4380	0.5760	0.1380

Table 2: MLE, B1, B2, B3 and B4 of TIITFIR model for (θ=0.8, α =0.5)

n	MLE	B1	90%			95%		
			B2	B3	B4	B2	B3	B4
30	0.8888	0.0324	0.5974	1.1802	0.5828	0.5416	1.2360	0.6944
	0.5621	0.0251	0.3385	0.7856	0.4471	0.2957	0.8284	0.5328
50	0.8611	0.0273	0.6454	1.0768	0.4313	0.6041	1.1181	0.5139
	0.5389	0.0131	0.3718	0.7060	0.3342	0.3398	0.7380	0.3982
100	0.8065	0.0090	0.6680	0.9451	0.2771	0.6415	0.9716	0.3301
	0.5024	0.0050	0.3907	0.6142	0.2235	0.3693	0.6356	0.2663
200	0.8130	0.0034	0.7144	0.9117	0.1973	0.6955	0.9306	0.2351
	0.5068	0.0024	0.4273	0.5863	0.1591	0.4120	0.6015	0.1895
500	0.7954	0.0011	0.7349	0.8560	0.1211	0.7233	0.8676	0.1443
	0.4936	0.0009	0.4443	0.5429	0.0986	0.4349	0.5523	0.1174

Table 3: MLE, B1, B2, B3 and B4 of TIITFIR model for (θ=0.8, α =0.8)

n	MLE	B1	90%			95%		
			B2	B3	B4	B2	B3	B4
30	0.8868	0.0428	0.5967	1.1768	0.5800	0.5412	1.2323	0.6911
	0.9072	0.0664	0.5468	1.2676	0.7208	0.4778	1.3367	0.8589
50	0.8492	0.0218	0.6375	1.0609	0.4235	0.5969	1.1015	0.5046
	0.8657	0.0389	0.5959	1.1355	0.5396	0.5443	1.1872	0.6429
	0.8175	0.0069	0.6763	0.9588	0.2825	0.6492	0.9858	0.3366

100	0.8284	0.0132	0.6446	1.0122	0.3676	0.6094	1.0474	0.4380
200	0.8161	0.0043	0.7168	0.9154	0.1986	0.6978	0.9344	0.2366
	0.8206	0.0068	0.6919	0.9492	0.2574	0.6672	0.9739	0.3066
500	0.7992	0.0012	0.7383	0.8601	0.1218	0.7266	0.8718	0.1452
	0.7937	0.0018	0.7146	0.8728	0.1582	0.6994	0.8879	0.1885

Table 4: MLE, B1, B2, B3 and B4 of TIITFIR model for (θ=1.2, α =0.5)

n	MLE	B1	90%			95%		
			B2	B3	B4	B2	B3	B4
30	1.4029	0.1838	0.8721	1.9337	1.0616	0.7705	2.0353	1.2649
	0.5715	0.0229	0.3671	0.7759	0.4088	0.3279	0.8150	0.4871
50	1.2258	0.0340	0.8844	1.5671	0.6828	0.8190	1.6325	0.8135
	0.5050	0.0076	0.3613	0.6488	0.2874	0.3338	0.6763	0.3425
100	1.2339	0.0246	0.9907	1.4771	0.4864	0.9441	1.5237	0.5796
	0.5128	0.0044	0.4097	0.6159	0.2062	0.3900	0.6356	0.2456
200	1.2087	0.0083	1.0420	1.3754	0.3334	1.0100	1.4073	0.3973
	0.5043	0.0019	0.4324	0.5762	0.1438	0.4186	0.5899	0.1713
500	1.2007	0.0058	1.0966	1.3049	0.2083	1.0766	1.3248	0.2482
	0.5011	0.0009	0.4559	0.5463	0.0904	0.4473	0.5550	0.1077

Table 5: MLE, B1, B2, B3 and B4 of TIITFIR model for (θ=1.2, α =0.8)

n	MLE	B1	90%			95%		
			B2	B3	B4	B2	B3	B4
30	1.2755	0.0738	0.8035	1.7475	0.9440	0.7132	1.8379	1.1247
	0.8590	0.0355	0.5436	1.1745	0.6309	0.4832	1.2349	0.7517
50	1.3052	0.0668	0.9343	1.6760	0.7417	0.8633	1.7470	0.8837
	0.8377	0.0184	0.6026	1.0727	0.4702	0.5576	1.1178	0.5602
100	1.2416	0.0321	0.9972	1.4860	0.4889	0.9504	1.5328	0.5825
	0.8263	0.0119	0.6608	0.9919	0.3310	0.6291	1.0236	0.3944
200	1.2533	0.0155	1.0789	1.4277	0.3488	1.0455	1.4611	0.4156
	0.8340	0.0066	0.7162	0.9518	0.2356	0.6936	0.9744	0.2807
500	1.2082	0.0022	1.1031	1.3133	0.2102	1.0830	1.3334	0.2505
	0.8009	0.0012	0.7287	0.8731	0.1444	0.7149	0.8869	0.1720

Table 6: MLE, B1, B2, B3 and B4 of TIITFIR model for (θ=2.0, α =0.5)

n	MLE	B1	90%			95%		
			B2	B3	B4	B2	B3	B4
30	2.3991	0.7301	1.3074	3.4908	2.1833	1.0984	3.6998	2.6014
	0.5461	0.0162	0.3679	0.7244	0.3565	0.3337	0.7585	0.4248
50	2.2350	0.3713	1.4695	3.0005	1.5310	1.3229	3.1471	1.8242
	0.5343	0.0095	0.3980	0.6707	0.2727	0.3718	0.6968	0.3250
100	2.0971	0.1092	1.6068	2.5874	0.9807	1.5129	2.6813	1.1685
	0.5094	0.0039	0.4170	0.6017	0.1847	0.3993	0.6194	0.2201
200	2.0111	0.0303	1.6849	2.3374	0.6525	1.6224	2.3999	0.7774
	0.5042	0.0012	0.4393	0.5692	0.1299	0.4268	0.5816	0.1548
500	2.0450	0.0135	1.8345	2.2555	0.4210	1.7942	2.2958	0.5016
	0.5081	0.0004	0.4668	0.5493	0.0825	0.4589	0.5572	0.0983

Table 7: MLE, B1, B2, B3 and B4 of TIITFIR model for (θ=2.0, α =0.8)

n	MLE	B1	90%			95%		
			B2	B3	B4	B2	B3	B4
30	2.1812	0.3513	1.2159	3.1465	1.9306	1.0311	3.3313	2.3003
	0.8348	0.0283	0.5571	1.1125	0.5553	0.5040	1.1656	0.6617
50	2.2678	0.3165	1.4896	3.0461	1.5564	1.3406	3.1951	1.8545
	0.8619	0.0226	0.6426	1.0811	0.4385	0.6007	1.1231	0.5224
100	2.1041	0.0906	1.6104	2.5979	0.9875	1.5158	2.6924	1.1766
	0.8293	0.0091	0.6787	0.9798	0.3011	0.6499	1.0087	0.3588
200	2.0263	0.0369	1.6962	2.3563	0.6601	1.6330	2.4195	0.7866
	0.8064	0.0031	0.7025	0.9103	0.2079	0.6826	0.9302	0.2477
500	2.0035	0.0122	1.7979	2.2092	0.4114	1.7585	2.2486	0.4901
	0.8041	0.0013	0.7385	0.8698	0.1313	0.7259	0.8824	0.1565

Table 8: MLE, B1, B2, B3 and B4 of TIITFIR model for (θ=2.0, α =1.2)

n	MLE	B1	90%			95%		
			B2	B3	B4	B2	B3	B4
30	2.2103	0.5561	1.2379	3.1827	1.9447	1.0517	3.3689	2.3171
	1.2282	0.0898	0.8237	1.6328	0.8091	0.7462	1.7102	0.9640
50	2.2835	0.2973	1.5007	3.0663	1.5656	1.3508	3.2162	1.8654
	1.2911	0.0438	0.9631	1.6191	0.6560	0.9003	1.6819	0.7816
	2.1281	0.1039	1.6293	2.6270	0.9977	1.5337	2.7225	1.1887

100	1.2544	0.0211	1.0276	1.4813	0.4537	0.9841	1.5247	0.5406
200	2.0224	0.0559	1.6918	2.3531	0.6613	1.6285	2.4164	0.7880
	1.2071	0.0128	1.0512	1.3629	0.3117	1.0214	1.3927	0.3714
500	2.0071	0.0140	1.8013	2.2129	0.4117	1.7618	2.2523	0.4905
	1.2021	0.0042	1.1040	1.3001	0.1961	1.0853	1.3189	0.2337

6. Modelling to Real Data

In this section, real data set is analyzed to illustrate the merit of TIITFIR distribution compared to some models; namely, IR, TIITLIR, *Frèchet* (F) distribution, *Marshall–Olkin Frèchet* (MOF) (Krishna *et al.* (2013)), *transmuted Frèchet* (TF) (Mahmoud and Mandouh (2013)),. Their density functions (for $x > 0$) are given in Table 9.

Table 9: The pdfs for some lifetime distributions

Distribution	The probability density function
F	$f_F(x;\theta,\sigma) = \theta\, \sigma^{\theta} x^{-(\theta+1)} e^{-\left(\frac{\sigma}{x}\right)^{\theta}}; \quad x, \theta, \sigma > 0.$
TIITLIR	$f_{TIITLIR}(x;\ \alpha,\theta) = 4\theta\alpha^2 x^{-3} e^{-2\left(\frac{\alpha}{x}\right)^2}\left[1-e^{-2\left(\frac{\alpha}{x}\right)^2}\right]^{\theta-1};\ x, \alpha, \theta > 0.$
TF	$f_{TF}(x;\alpha,\beta,b) = \beta\alpha^{\beta} x^{-\beta-1} exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right]\left\{1+b-2b\ exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right]\right\}; x,\alpha,\beta,b > 0.$
MOF	$f_{MOF}(x;\alpha,\beta,a) = a\beta\alpha^{\beta} x^{-\beta-1} exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right]\left\{a + (1-a) exp\left[-\left(\frac{\alpha}{x}\right)^{\beta}\right]\right\}^{-2}; x,\alpha,\beta,a > 0$

We obtain the ML estimates, and *standard errors* (SEs) of the model parameters. To compare the distribution models, we consider criteria like; *minus of log-likelihood function* (-2lnL), *Akaike information criterion* (AIC), the *correct Akaike information criterion* (CAIC), *Bayesian information criterion* (BIC), *Hannan-Quinn information criterion* (HQIC). However, the better distribution corresponds to the smaller values of -2 lnL, AIC, CAIC, BIC and HQIC.

The dataset comprises of 100 observations of carbon fiber breaking stress (in Gba) provided by Nichols and Padgett (2006). Table 10 displays the ML estimates and SEs for the ten models. Table 11 displays the values of -2lnL, AIC, BIC, HQIC, and CAIC.

Table 10: The ML estimates and SEs of the model parameters for the data set

Model	ML estimates and SEs	
TIITFIR(α,θ)	2.805 (0.41998)	0.638 (0.068)

TIITLIR	1.341 (0.092)	1.15 (0.167)	
IR(α)	3.269 (0.3269)		
F(θ, σ)	1.8705 (0.112)	1.7766 (0.113)	
TF(α, β, b)	1.9315 (0.097)	1.7435 (0.076)	0.0819 (0.198)
MOF(α, β, a)	2.3066 (0.498)	1.5796 (0.16)	0.5988 (0.3091)

Table 11: The values of -2LnL, AIC, BIC, HQIC and CAIC for the data set

Model	Goodness of fit criteria				
	$-2\hat{\ell}$	AIC	BIC	HQIC	CAIC
TIITFIR	315.393	319.393	319.393	321.502	319.517
TIITLIR	348.097	352.097	352.097	354.205	352.22
IR	349.007	351.007	351.007	352.061	351.048
F	344.3	348.3	353.5	350.4	348.4
TF	344.5	350.5	358.3	353.6	350.7
MOF	345.3	351.3	359.1	354.5	351.6

We discover that the TIITFIR distribution with three parameters fits better than the five models. It has the lowest AIC, BIC, HQIC, and CAIC values among the candidates studied here.

Figures 5, 6, and 7 show empirical pdf, cdf, and sf plots from the data set for the TIITFIR, TIITLIR, IR, F, TF, and MOF models, respectively.

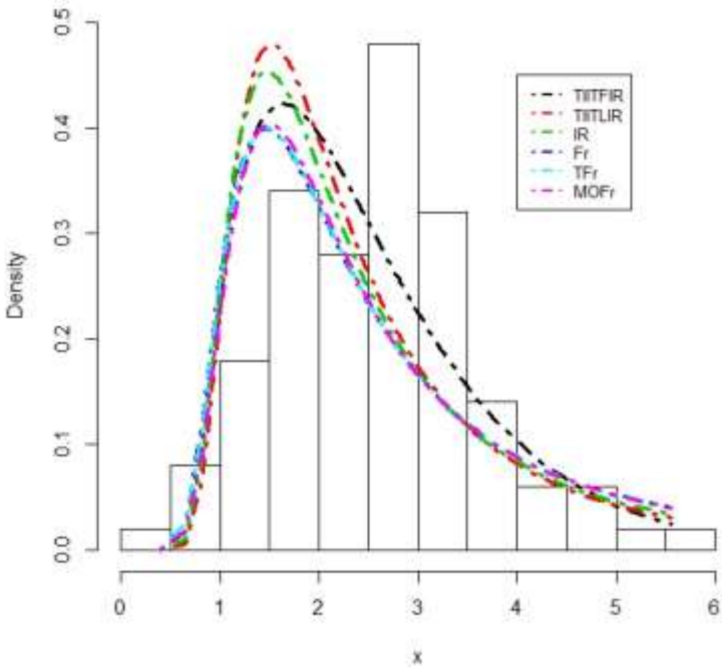

Figure 5. Estimated pdf of models for the data set

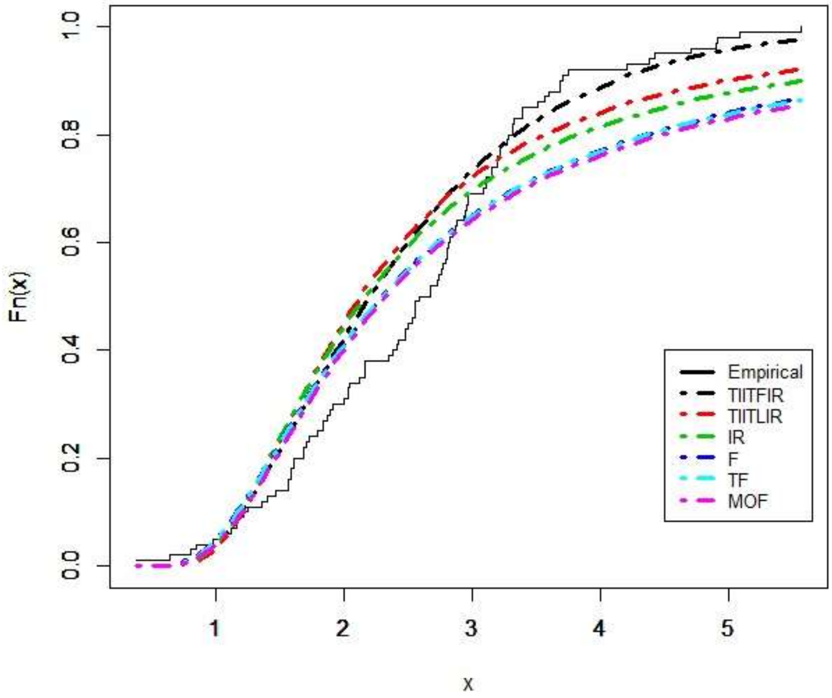

Figure 6. Estimated cdf of models for the data set

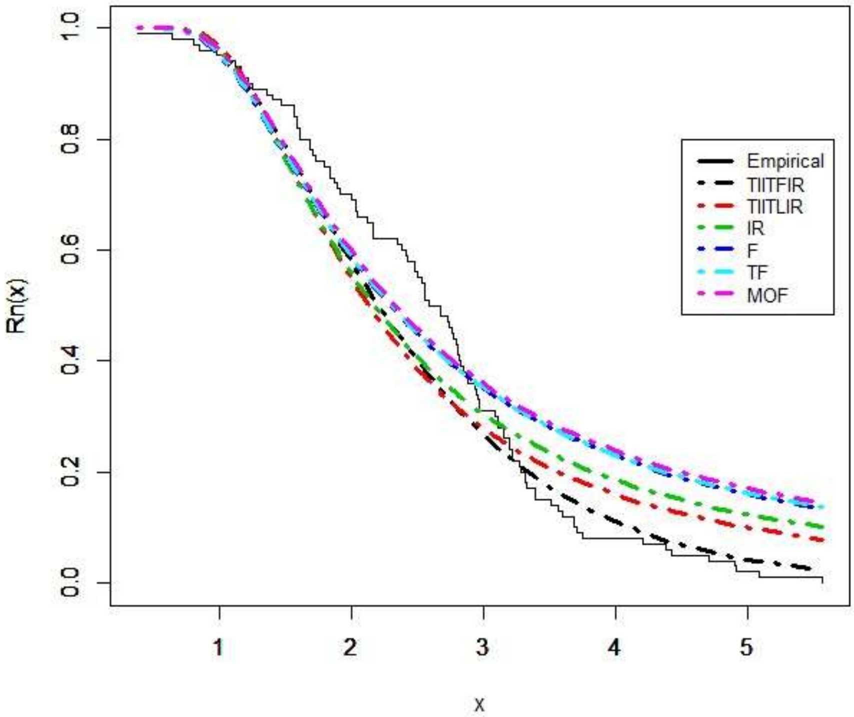

Figure 7. Estimated sf of models for the data set

Figures 5, 6, and 7 show that the TIITLIR distribution fits better than the competing models.

7. Conclusion

Inside this study, we present the TIITFIR model, a novel two-parameter lifespan model. TIITFIR's pdf may be described as a linear mixture of IR pdfs. Some of its statistical features are investigated. We investigate ML estimation. The accuracy and performance of estimations are evaluated using simulation results. Using an actual data set, the suggested model outperforms five other competing models in terms of fit.

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