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Van Hiele and GeoGebra model. An Analysis from Variational Thinking in Basic Education Students

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Abstract

The article describes the use of Geogebra within the framework of the Van Hiele model for the development of the learning process of the concept of function in basic education students. The methodology is qualitative action research with quantitative support. To this end, a didactic sequence was implemented with situations of variation and changes in the records of representation of the concept of function, which was analyzed, from the qualitative through direct observation and interviews with students, and from the quantitative, through a pretest design - posttest. Students had the opportunity to experience another way of learning using dynamic environment such as Geogebra. It is concluded that the levels of reasoning of Van Hiele and the Geogebra software are adequate for the study of functions, because it allowed learning situations where students evidenced the analysis of graphs, evaluation of algebraic expressions and changes of representation in adynamic and interactive environment.

Key words: Function, Van Hiele, Learning, Geogebra, Variational Thinking.

1. INTRODUCTION

The learning difficulties evidenced by students in international tests such as PISA show their low levelof mathematics (Organization for Economic Cooperation and Development [OECD], 2012). Castro et al. (2014) show that in 2006 and 2012, most of the students in Colombia evaluated have been below the average established by the OECD. Likewise, according to TERCE (Regional Comparative and Explanatory Study), 3rd and 6th grade students who presented the Mathematics test show results below the regional average in most domains and processes (United Nations Educational, Scientific and Cultural Organization[UNESCO], 2016). In the same sense, the national tests (SABER mathematics of 3rd, 5th and 9th grade) show significant differences between public and private education, with the entities of iciales reflecting low percentages (Colombian Institute for the Evaluation of Education [ICFES], 2016).

On the other hand, Vargas and Gamboa (2013), consider that geometry is one of the branches of mathematics that is most important for humanity and its development. Among the most relevant contributions to the teaching of geometry are the contributions of Pierre Marie Van Hiele and Dina Van Hiele-Geldof, in their 1957 work Structure and Insight : A theory of mathematics education (Burger & Shaughnessy , 1986), which laid the foundations for a model that on the one hand describes the different types of geometric reasoning of students, ranging from intuitive to formal and abstract reasoning and on the other, a description of how a teacher can organize the activity in his classes so that students can reach the higher level of reasoning (Guillén, 2004), five learning phases that constitute a scheme for organizing teaching (Antezana et al., 2020).

In addition, according to the Curricular Guidelines of mathematics (Mineducación, 1998), the Van Hiele Model is a method for the construction of geometric thinking, which, despite not being recent, has not lost any validity and its main ideas conform to current didactics (Fouz & De Donosti , 2005). In this sense, it provides guidelines for the curricular organization in geometry in the area of mathematics of the different educational levels (Gutiérrez, 1998). Likewise, in addition to offering an interesting way to identify the characteristics or levels of reasoning in which it progresses until reaching the highest level of learning, it allows evaluating the quality of its levels of reasoning (Vargas & Gamboa, 2013; Falconí-Procel, 2021), this allows students to be guided so that they can reach the higher level of reasoning (Gutiérrez & Jaime, 1998a; 1998b), which allows them to decide the rigor of their classes (Gutiérrez, 1994).). Finally, some studiesthat implement the model, among which can be mentioned (Huerta, 1999; Gualdrón, 2014; Rodriguez-Perez, 2015; Santafé, 2019) among others.

According to Gutiérrez and Jaime (1998), Van Hiele's reasoning model is not exclusive to the study of the concepts of Geometry, but is applicable to any mathematical concept (Llorens & Prat Villar, 2015). The reasons given are fundamental in thestudy to implement the Van Hiele model with the approach of learning situations to develop in students the levels of reasoning and learning phases as described by Fouz and De Donosti (2005).

Tsister 1 Van Hideen levels of reasoning.

Denomination	**Description**
Level 0: Visualization or Recognition	1) Objects are perceived in their entirety as a unit, without differentiating their attributes and components. 2) They are described by their physical appearance through purely visual descriptions and resembling familiar elements of the environment (it looks like a wheel,it is like a window, etc.). There is no basic geometric language to call the figures by their correct name. 3) Do not explicitly recognize components and properties of the object work motive
Level 1: Analysis	1) The components and property(necessary conditions) of objects and figures are perceived. They obtain this both from observation and experimentation. 2) In an informal way they can describe the figures by their properties, but not by relating some properties to others or somefiguras to others. As many definitions in geometry are elaborated from properties they cannot elaborate definitions. 3) Experimenting with shapes or objects can establish new properties. 4) However, they do not classify objects and propertieson the basis of their properties.

Level 2: Sorting or Classification	1) The figures are described in a formal way, that is, the necessary and sufficient conditions that must be met are indicated. This is important because it involves understanding the meaning of definitions, their role within geometry and the requirements theyalways require. 2) They perform logical classifications in a formal way since the level of their mathematical reasoning is already started. This means that they recognize how some properties derive from others, establishing relationships between properties and the consequencesof those relationships. 3) The demonstrations continue, but, in most cases, they do not understand them in terms of their structure. This is because their level of logical reasoning is able to follow individual steps of reasoning, but not todo so as a whole. This lack prevents them from grasping the axiomatic nature of Geometry.
Level 3: Formal Deduction	1) At this level, logical and formal deductions and demonstrations are already made, seeing their need to justify the propositions proposed. 2) The relationships between properties are understood and managed and formalized in axiomatic systems, so the axiomatic nature of mathematics is already understood. 3) It is understood how the same results can be reached based on different propositions or premises, which allows us to understand that different forms of demonstrations can be made to obtainthe same result. It is clear that, acquired this level, having a high level of logical reasoning, you have a globalizing vision of Mathematics.
Level 4: Rigor	1) The existence of different axiomatic systems is known and can be analyzed and compared allowing to compare different geometries. 2) Geometry can be worked on abstractly without the need for concrete examples, reaching the highest level of mathematical rigor.

Table 2 Teaching phases of the Van Hidee model.

Phase	**Description**
Questions – information	This phase is oral and through the appropriate questions it is about determining the starting point of the students and the way forward of the following activities. It can be done through a test or individualized questions using activities of the starting level. It should be noted that many times the level is not marked so much by the question as the answer, that is, we design a question thinking about a specific level and, the answer received, can point us to a different level from the one initially thought.
Targeted guidance	This is where the importance of the teacher's didactic capacity will be most needed. From their experience they point out that the performance of the students (optimal results compared to time spent) is not good if there are not a series of concrete activities, well sequenced, so that the students discover, understand, assimilate, apply, etc. the ideas, concepts, properties, relationships, among others, that will be the reason for their learning at that level.

Explanation (explicit)	It is a phase of interaction (exchange of ideas and experiences) between students and in which the role of the teacher is reduced in terms of new content and, however, his action is aimed at correcting the language of the students as required at that level. The interaction between students is important since it forces them to order their ideas, analyze them and express them in a way that is understandable to others.
Free orientation	More complex activities appear fundamentally related to applying the previously acquired, both with respect to content and the necessary language. These activities should be sufficiently open, ideally open problems, so that they can be approached in different ways or can be of several valid answers according to the interpretation of the statement. This idea forces them to a greater need to justify their answers using increasingly powerful reasoning and language.
Integration.	The first important idea is that, in this phase, new contents are not worked on, but only those already worked on are synthesized. It is about creating an internal network of learned or improved knowledge that replaces the one you already had. As a final ideawe can point out how in this structure of activities can be perfectly integrated recovery activities for students who present some delay in the acquisition of geometric knowledge and, on the other hand, adequately redoing the groups to deepen something more with those students of better performance, Although the evaluation activities have not been explicit, They would also be easily integrated into this structure of activities.

GeoGebra is a free, dynamic software that allows a pleasant environment for both the student and the teacher, where geometry, algebra and calculus are dynamically combined (Hohenwarter & Fuchs, 2004). In this sense, Ruiz et al. (2013) describe the program as a didactic tool due to the environment it offers and García (2011) where it expresses the attributes such as constructivity, interactivity and ease of use and speed of response and with its advantages, as a motivated function. On the dynamic environment GeoGebra there are several studies that account for its use as didactic mediation in the teaching-learning process of mathematics, within the classroom, provides certain advantages to the teacher and the student as a form of representation, visualization of certain concepts or procedures (Ruiz et al., 2013), in addition to helping to improve attitudes towards mathematics due to the taste and confidence that they deposited in their use to geometric contents (Hernández et al., 2022; Garcia et al., 2021).

In that sense, on the use of technological tools and use of software as mediations in the teaching-learning process since its use has made more accessible and important for studentss topics of geometry, probability, statistics and algebra which expands the field of inquiry on which act the cognitive structures that are had, enrich the curriculum with the new associated pragmatics and lead it to evolve (Mineducación, 1998). Some studies that can be mentioned on the relationship of Geogebra with the Van Hiele model are the following (Acevedo et al., 2008; Cáceres, 2017; Antezana et al., 2020; Linares, 2020; Calderón-Gualdrón & Londoño-Cano, 2021) among others.

1.1 Approaches to learning as cognitive development

Learning is a change in the meaning of experience (Novak & Gowin, 1984) that is relatively permanent from the behavior that occurs as a result of practice (Shunck, 2012), and incorporating it into other knowledge previously accumulated by the same process (Díaz & Hernández, 2002). Constructivism is an educational approach that integrates various psychological theories of learning and the epistemology of knowledge construction (Pozo, 1997), especially Ausubel's theory (1976; 2002) on meaningful learning that establishes that the construction of new knowledge that highlights the role of concepts, the relationships between them; prior knowledge and language to shape, codify, and acquire new meanings (Novak, 1988) that delves into the meaning and sense of student learning (Ausubel et al., 1983), its nature; its conditions for it to occur; in its results and evaluation (Ausubel, 1976).

1.2 Variational thinking and algebraic systems

According to the curricular guidelines of mathematics (Mineducación, 1998) mention variational thinking and algebraic systems as one of the axes of basic knowledge in the teaching and learning of mathematics having as conceptual nuclei of variation such as function as dependence and function models. The same function can be represented in all possible ways, including, sometimes, it is convenient to use several representations of thesame function to have a more complete knowledge of it. Keep in mind that certain functions are described more naturally with one method than with another. In the process of intervention in the classroom, types of representaciones of a function will be used. Vivas (2010) describe five representation systems relevant to the description of a function: Verbal (description with words), Symbolic (equation or algebraic expression), Visual (graph, sagittal diagram, Cartesian diagram), numerical (table of values) and geometric. Each of these representations allows to express a phenomenon of change, a dependence between variables.

According to the above, the proposed objective was to describe the application of the Van Hiele Model and Geogebra in the learning of basic education students to develop variational thinking through the concept of function. The interpretation of the Van Hiele model in a mathematical object such as the function, which is not directly a geometric concept, will be described with the levels as do Gutiérrez and Jaime (1998a), Fouz and De Donosti (2005), Vargas and Gamboa (2013), Fuentes et al. (2015), Aravena et al. (2016) among others

2. METHODS

This study was based on a territorial approach withresearch-action methodology, and is supported through: observations in the classroom, opinions and participation of students, with quantitative support in the analysis of diagnostic test results and purpose.

2.1 Research subjects.

It corresponds to students of basic secondary education of the ninth grade of a public educational institution in the municipality of Chinácota, Norte de Santander – Colombia.

2.2 Instruments

Among the instruments forcollecting information is the direct observation of students in the classroom, the pretest and posttest. On observation, Stake (1999), expresses that they lead the researcher towards a better understanding of the case, being a methodor collection of information that consists of the systematic, valid and reliable record of observable behaviors and situations, through categories and subcategories (Hernández et al., 2014). In the same order, on the pre-test, post-test and the different sessions of the intervention, they were reviewed one by one, by the teachers of the mathematics area, and relied on experts, whose contributions helped to improve the sessions that were not so clear to the students.

2.3 Intervention

In the intervention, Van hiele's theoretical model is interrelated, which is constituted as the construct that bases this work, from the perspective of its initial levels. The concept of function as an object of study and GeoGebra as adidactic mediation for the learning of mathematics in a dynamic way. The intervention is constituted as a pedagogical classroom project, which takes into account the components of the curriculum, based on the needs of the students (Romero & Montoya, 2008). The intervention for the approach to the concept of function was constituted by the sessions of Diagnosis, Concept of function, Domain and Rank, and Types of function, which contained activities and tasks, with a duration between 2 to 3 hours of work and that correspond to strategies for the strengthening of numerical-variational thinking that is the lowest performance.

3. RESULTS AND DISCUSSION

To interpret the results, Table 1 describes the categories of teaching and learning factors below.

Table 3

Final category of teaching and learning factors.

Factor	Category - Code	Subcategory - Code	Subcategory - Code
Teaching	Function [F]	Concept of function [F1]	Relationship [F.1.1]
			Independent variable [F.1.2]
			Dependent variable [F.1.2]
		Elements of a function [F.2]	Domain [F.2.1]
			Co-domain [F.2.2]
			Range [F.2.3]
			Growth [F.2.5]
			Degrowth [F.2.6]
		Representation records [F.3]	Natural language register [F.3.1]
			Algebraic symbolic register [F.3.2]
			Graphic record [F.3.3]
			Tabular register [F.3.4]
		Sequencing [F.4]	Changing the context [F.4.1]
			Translating representation records [F.4.2]
			Dealing with registration [F.4.3]
			Using GeoGebra [F.4.4]
	Didactic Intervention [ID]	Clarify [ID.1]	
		Fix [ID.2]	
		Feedback [ID.3]	
		Strengthen [ID.4]	
		Motivate [ID.5]	
Learning	Competences [C]	Communication [C 1]	Graphics Features [C.1.1]

		Characteristics of functions [C.1.2]
		Representing functions [C.1.3]
		Evaluate expressions [C.1.4]
		Modeling Variation Situations [C.1.5]
		Translate records [C.1.6]
	Technological [C.2]	Using GeoGebra [C.2.1]
	Attitudinal [C.3]	Positive attitude [C.3.1]
		Interest [C.3.2]
		Motivation [C.3.3]
		Attention [C.3.4]
		Participation [C.3.5]
Meaningful Learning [AS]	Production of ideas [AS.1]	
	Reasoning and observation [AS.2]	
	Prior knowledge [AS.3]	
	Recognition of the situation in context [AS.4]	

3.1 Results of the diagnostic test (pretest)

As a result of the application of the diagnostic test (pretest), it was evidenced that the previous knowledge of the students, before questions of situations that have to do with the concept of relationship and function, are in the level 1 of Van Hidee reasoning, very close to what was expressed by Fouz and De Donosti (2005) of merely visual descriptions and resembling familiar elements of the environment. Also, it was verified that the presence of the teacher and the collaboration between the students is necessary, especially in analysis activities, where the answers were not significant, so it is evident that the student alone does not build his knowledge, but does so thanks to mediation and interaction with others, and in the classroom environment are the teacher and his classmates (Parra, 2014; Prada et al., 2022).

3.2 Results of the intervention

During the implementation of the intervention approach to the concept of function, when inquiring about what is meant by relationship [F.1.1] the category [AS.4] recognition of the context situation is achieved, by classifying it according to its characteristic, they show Van Hijale level 1 reasoning. Some students were able to describe that a relationship is a set of ordered pairs, formed from the correspondence between the elements of two sets" (Becerra, 2004).

When proposing questionsabout tabular representation [F.3.4], the categories model situations of variation [C.1.5] and reasoning and observation [AS.2] emerge, which shows the construction of knowledge, which evidences significant learning (Moreira, 1997) hence the importance of the student's interest in assuming the challenge of learning, through a material that is interesting (Flores-Espejo, 2018).

By presenting situations on Cartesian graphic record [F.3.3], students achieve learner categories andmodel situations of variation [C.1.5] and translate records [C.1.6] with answers that can be considered in Van Hiele's level 2. Castiblanco et al. (2004), affirms the importance of graphs and tables to model situations of change and the importance of exercising the translations from one to another of the different representations of a function, being this type of activities environments to promote in the students the development observe and describe situations of variation, express and traducir between different registers [C.1.6] of representation [F.3]; [F.4.2]; [C.1.1]; [C.1.3]. When proposing to change from a graphic register to an algebraic register, participants require reinforcement [ID.4] to assimilate this change, in this way theyevaluate an algebraic expression [C.1.4]. Students alone will not be able to advance to the next level of reasoning, but the use of strategies that the teacher presents in an appropriate way, achieves the advance in the level of reasoning that is intended (Salvador, 1994).

The use of the GeoGebra program [C.2.1]; [F.4.4] is very practical, which is reflected in tasks in which students had no problems entering expressions, which are displayed in the work area. Thus, they have the opportunity to study the elements of a function, where they verify whether a curve in the plane is a function [F.2]. Thus Geogebra becomes a mediator of learning, by shared meanings (Labarrere, 2008). In addition, the software allows the interaction between the different registers (algebraic, graphic, tabular), which was evidenced when students had to make a replica of the graphs [F.3] and verify whether or not it is a function by translating the record into graphic form [F.4.2]. The interactive environment of GeoGebra allows the student to raise their reasoning of variational thinking in addition to creating a pleasant environment, predisposition to perform the tasks and different activities that are proposed [C.2.1].

Another important activityto comment has to do with sequencing using the Geogebra software [F.4.4] where they created examples of functions using the drag tool, as well as the use of sliders to appreciate how the value of the function changes, calleddeformation pr oceso (Gutiérrez & Prieto, 2015), all these activities in a guided way so that they can apply it in the following activities.

Finally, in the intervention a situation of variation between two magnitudes was proposed, a situation with a graphic[F.3.3] with a series of tasks that involve observation, analysis and reasoning to draw conclusions. The answers given by the students show the ability to describe the characterization corresponding to the growth, decreaseor constant of the situation and have been able to describe, interpret, predict its consequences, quantify it and model it from a situation of variation represented in a graph [AS.2]; [C.1.1]; [C.1.5], are the characteristics of variational thinking" (Castiblanco et al., 2004).

It was possible to mesh the Van Hiele Model with GeoGebra and its corresponding levels of reasoning especially situations of level 1 and 2 of reasoning, where it is stated that the thinking of the second level no is possible without the basic level. (Van Hiele, 1986). And this is where teacher mediation is necessary so that students can move to the next level of reasoning, in addition to which it is necessary that students are motivated, and work collaboratively, since Geogebra is effective in improving students' attitudes towards mathematics, due to the taste and trust they placed in its use for the study of content (García et al., 2021).

On the competences [C] that the students had to develop, the way of expressing functions between different registers of representation, between verbal, graphic and tabular languages, was revealed. The interaction of the teacher and the students [ID] is what succeeds in fosteringthe learning process. Students alone will not deeply understand the concepts that need to be studied.

3.3 Results of the final evaluation (posttest)

In the final evaluation, several situations of variation [C.1.5] are presented, which show time as the independent variable [F.1.2], which helps students to find meaning in the study of functions and the development of variational thinking, from problematic situations whose scenarios are those referring tophenomena of change and variation of practical life (Mineducación , 1998). Graphs and tables are necessary to model situations of change and the importance of exercising translations from one to another of the different representations of a function (Castiblanco et al., 2004).

3.4 Pretest and Posttest comparison

It is established that if there is a difference between the results of the diagnosis where the students were at a level 1 of reasoning, and that thanks to the activities and learning situations of the intervention, the results of the posttest evidence that most students move to a level 2 of reasoning, which was observed when students evaluate an algebraic expression, translate representation registers, graph a function, and extract its elements and characteristics (Prada et al., 2017; Arciniegas et al., 2018).

5. CONCLUSIONS

The diagnostic test allowed to characterize the preconceptions and concepts related to the functions and that thanks to the Van Hiele model it was evident that the students made level 1 reasoning. From these results, a sequence and didactic intervention was designed that allowed students to move to reasoning level 2, in a deductive way, moving from observation or recognition tasks to analysis and classification. For the design of the intervention, the use of GeoGebra was taken into account for tasks such as changes in register representation, through sequences such as context change, translation of representation records, among others, to developvariational thinking.

It was established that the Van Hiele Reasoning Model is appropriate for the learning of students in the concept of function, through activities and tasks in the dynamic environment of Geogebra on aspects such as the elements of the function, its characteristics, classification, from graphic situations that involve skills such as: to the analysis of graphs, model situations of variation, evaluate an algebraic expression, translate representations andothers.

The use of strategies based on the Van Hiele model that uses Geogebra as a mediation in the construction of the concept of function and development of variational thinking, in students of basic education, was approached from three aspects. The first, related to the design of a pedagogical classroom project relevant and coherent for learning as recommended by the Ministry of National Education through its curricular guidelines, the second aspect, refers to the USor ICT, where digital skills were revealed through mediation such as Geogebra , and the last aspect, has to do with the attitudinal component of the students that is reflected in the motivation, interest, and positive attitude, towards the study of mathematical concepts.

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